A
GENERAL TREATISE
OF
Menſuration:

CONTAINING

Many uſeful and neceſſary Improvements.

Compoſed for the Benefit

OF

ARTIFICERS, BUILDERS, MEASURERS,
SURVEYORS, GAUGERS, FARMERS,
GENTLEMEN, YOUNG STUDENTS, *&c.*

The Whole being intended as an eaſy Introduction
to ſeveral Parts of the

Mathematics.

By J. ROBERTSON, F. R. S.

The SECOND EDITION, with many ADDITIONS.

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BRITISH MUSEUM

To William Jones, *Eſq*;

Fellow *of the* Royal-Society.

Sir,

As the publication of the former edition of this treatiſe, procured me the honour of your acquaintance, the conſequences whereof have been one continued courſe of favours, which you have with the utmoſt generoſity conferred on me; among theſe may be juſtly ranked the free acceſs you have conſtantly allowed me to your books and papers, and alſo your friendly advice whenever deſired: Therefore, no place more proper than in the preſent work, to return you my ſincere thanks in this public manner.

AND, as many of the improvements in this impreſſion, are owing to your communications; ſo I beg you will accept of this ſmall offering, and grant it your protection; this I hope for, becauſe that candour and benevolence, which you ſo frequently, and on ſo many occaſions, exert to promote whatever appears deſign'd for public, or for private uſe, is no leſs conſpicuous, than is your eminent knowledge in every branch of ſpeculative, and practical mathematics.

I am,

SIR,

Your moſt obliged,

And moſt humble Servant,

John Robertſon.

THE

PERFACE.

MENSURATION, if conſidered in its utmoſt extent, would include all the branches of practical mathematicks; and as theſe depend on the principles delivered in the ſpeculative parts, therefore a Treatiſe *of* Menſuration *would not be an improper title to a ſyſtem or courſe of mathematical ſciences; for when theſe are applied to practical uſes, there ſeem to be few things beſide the meaſuring of lengths, ſuperficies, ſolids, angles, forces, motion, duration and chance: But as cuſtom has reſtrain'd the notion of menſuration to that of finding the lengths of lines, the ſuperficial and ſolid content of figures; therefore the reader is to expect, in the following ſheets, no more than what relates to the common acceptation of the word as now explain'd.*

About ten years ago, the author being deſirous of putting together a few papers on Menſuration *for the uſe of his pupils; to that purpoſe he* *he peruſed the ſeveral treatiſes on that head; and conceiving them to be defective, either in matter or method, this induced him then to compoſe, what he thought, a new ſyſtem, diſpoſed in a quite different order from any of thoſe that were done before; which being approv'd of by ſeveral good judges, he publiſhed it; and the impreſſion being ſometime ſince ſold off, he reviſed the whole; made ſo many alterations and additions thereto, that he was in doubt whether to call it a ſecond edition, or a new work.*

This book is divided into three parts, and is preceded by an introduction containing the doctrine of decimal fractions, and of duodecimal arithmetic; theſe are not here prefix'd meerly for their uſe in computing the ſuperficies and ſolidities of figures, for many other parts of the mathematics have the ſame claim; but becauſe herein are ſeveral articles not very common, and which the young ſtudent might, perhaps, find quite neceſſary in reading the other parts of this treatiſe.

The firſt part contains the manner of computing the areas of right lined and circular plane figures.

The ſecond part contains rules for meaſuring the contents of ſolids comprehended under right lined and circular figures.

The third part ſhews how the ſuperficial contents of thoſe figures commonly call'd conic ſections are to be computed; and alſo, the ſurfaces and contents of ſeveral ſolids generated by the motion of ſuch figures about certain right lines: To which are added, many things relating to the ſubject of Menſuration, *the particulars of which, are enumerated in the contents.*

The whole is illuſtrated with a great variety of, what is apprehended to be, uſeful examples; which are ſo contrived as to ſerve for exerciſes to ſeveral of the preceding propoſitions, and adapted to ſuch uſes as often occur in the common affairs of life: In thoſe which relate to artificers works concern'd in building, the operations are generally perform'd, both decimally and duodecimally: Alſo, the cuſtoms and allowances in the works of ſuch artificers, are inſerted in their proper places.

The reader will here find, not only what is to be met with in other books on the ſame ſubject, and what are diſperſed in miſcellaneous works, but many things, perhaps, entirely new; and others (not very common,) are ſo diſpoſed as to be much more practicable and uſeful, than they appear'd to be, in the form originally given them.

As this book is chiefly intended for ſuch perſons as are employ'd in practical buſineſs, therefore

fore the demonſtrations of the rules are omitted; referring more inquiſitive readers to the elementary books of Geometry: Nevertheleſs, the learner will here meet with an eaſy introduction to ſeveral parts of the mathematics; and thoſe who have made farther advances, will, at leaſt, find in this, a common-place-book for many rules, which may not be of ſufficient importance to burthen their memory with.

Upon the whole, the author has ſpared no pains to make it generally uſeful, and therefore, if any overſight has eſcaped him, either in the preſs or otherwiſe, among ſuch a multitude of articles, he hopes the candid reader will generouſly excuſe him; his view being never to find fault with others, but of endeavouring as much as in him lies, to promote theſe Studies, ſo highly beneficial to mankind.

October 29. 1747.

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THE

THE INTRODUCTION.

Containing the DOCTRINE OF *DECIMAL FRACTIONS.*

SECTION I.

I. ANY thing, conſidered as one, is called an *unit*. As one yard, one pound, one gallon, *&c.* are all units, of their reſpective kinds.

But one yard, one pound, one gallon, *&c.* may each be conſidered as compos'd of ſeveral leſſer quantities; then are theſe component things, conſidered as *parts* of their reſpective units.

Thus one yard may be conceived to conſiſt of 36 inches, or of 3 feet, or of four quarters; then are inches, feet and quarters, taken as parts of a yard: *&c.*

II. A fraction is ſome part, or parts of an *unit*.

Fractions ariſe from Diviſion, and are expreſſed by writing the Diviſor under the Remainder, with a Line drawn between them.

Thus if 38 was to be divided by 6, the quotient would be leſs than 7, and more than 6; that is, it would be 6 $\frac{2}{6}$.

III. In a fraction thus conſtituted, the number below the line is called the *denominator*, and ſhews into how many parts the unit is ſuppoſed to be divided: The number above the line is called the *numerator*, and ſhews the value of the fraction in parts of the denominator: Expreſſions of this kind are called *vulgar fractions*.

When the denominators of vulgar fractions conſiſt of ſeveral digits, their management is attended with ſome trouble; to obviate which, the following method will chiefly be uſed in this Treatiſe.

IV. Fractions, whoſe denominators conſiſt of unity with one, or more cyphers, are called *decimals, or decimal fractions.*

V. Decimal fractions are commonly wrote without their denominator; theſe being always underſtood to conſiſt of an unit, with as many cyphers annexed to the right-hand, as the decimal fraction (or numerator) has places: and to diſtinguiſh integral places from fractional ones, the latter have a point, or comma, ſet before the left-hand place.

Thus 0,6 is underſtood to be $\frac{6}{10}$;
And 0,47 - - - - $\frac{47}{100}$;
And 0,763218 - - - $\frac{763218}{1000000}$;
And 246,385 - - - $246\frac{385}{1000}$
&c.

VI. A finite decimal, is that which ends at a certain number of places; but an infinite, that which no where ends.

VII. A *recurring*, or *circulating decimal*, is that wherein one or more figures are continually repeated.

Thus 42,387666, *&c.* is call'd a ſingle circulate or recurring decimal.

And 285,264264264, *&c.* is call'd a compound recurring decimal.

VIII. The firſt place next the mark of diſtinction in any decimal expreſſion, is called the place of primes; and the following places are called ſeconds, thirds, fourths, *&c.*

IX. Cyphers to the right-hand of decimals, neither increaſe nor decreaſe their value; but cyphers between the ſeparating point and the digits of the decimal, diminiſh the value.

Thus 0,5, is $\frac{5}{10}$.
But 0,05, is $\frac{5}{100}$.
And 0,005, is $\frac{5}{1000}$, *&c.*

The like is to be underſtood in other decimals.

X. In any mixed or fractional number, if the mark of diſtinction be removed one, two, three, *&c.* places to the right-hand, then every place in that number will be 10, 100, 1000, *&c.* times greater than it was before. But if the mark be removed towards the left-hand, then every place will be diminiſhed in the ſame manner.

Thus	And
357,984	357,948
3579,84	35,7948
35798,4	3,57948
&c. increaſing	*&c.* decreaſing

Note, A number conſiſting of integral and fractional places, is called a mixed number.

XI. Every finite decimal may be conſidered as infinite, by making cyphers to recur. For they do not alter the value of the decimal.

XII. Any decimal expreſſion may be continued at pleaſure, by repeating the circulating figure or figures.

XIII. In all operations, if the reſult conſiſts of ſeveral nines, reject them, and make the next ſuperiour place an unit more; thus for 7,23999, *&c.* write 7,24.

XIV. In all circulating numbers, daſh the firſt and laſt of the recurring digits, omitting the intermediate places; thus, $4{,}28\cancel{3}$ or $7{,}\cancel{5}4\cancel{3}\cancel{5}4\cancel{3}\cancel{5}4\cancel{3}$, *&c.*

SECTION II.

REDUCTION.

OR the methods uſed to bring any vulgar fraction, or an expreſſion of different denominations to its equivalent decimal value: Or any decimal expreſſions, to its value in different denominations.

CASE I.

To reduce a vulgar fraction, to its equivalent decimal one.

RULE.

RULE.

Divide the numerator (with as many cyphers annexed, as may be neceſſary,) by the denominator; and the quotient will be the decimal ſought.

Note, For every cypher uſed with the numerator, there muſt be a cypher, or digit, in the decimal expreſſion found in the quotient; and the comma, or mark of diſtinction, muſt be ſet on the left-hand thereof.

EXAMPLES.

I. $\frac{1}{4}$ is equal to 0,25;
II. $\frac{1}{2}$ - - - 0.5
III. $\frac{3}{4}$ - - 0,75
IV. $\frac{5}{8}$ - - - - 0,625
V. $\frac{7}{12}$ - - - $0{,}58\dot{3}$.

From the laſt example, it will be eaſy to conceive how circulates are generated; and alſo, to ſee that 3 would continually repeat.

What is the decimal fraction equal to $\frac{9}{56}$*?*

```
56)9,00000000(0,16071428, &c.
   340
     400
        80
       240
        160
         480
          32
```

VII. *What is the decimal fraction equal to* $\frac{17}{286}$?

286) $17{,}000000\dot{0}$ (0,0594405, *&c.*

2700

1260

1160

1600

170

VIII. *What is the decimal fraction equal to* $\frac{217}{2495}$?

2495) $217{,}\dot{0}\dot{0}$ (0,08697394, *&c.*

$1740\dot{0}$

$2430\dot{0}$

$1845\dot{0}$

$985\dot{0}$

$2365\dot{0}$

$1195\dot{0}$

1970

IX. *What is the decimal fraction equal to* $\frac{83}{9768}$?

9768)83,000(0,008497133, *&c.*

48560

94880

69680

13040

32720

34160

4856

In Example VII. may be ſeen the generation of a compound recurring decimal: For after 7 cyphers are uſed, the remainder 170, with another cypher annexed, is equal to the number began with; conſequently the ſame figures, *viz.* 594405, will repeat in the quotient; and after them, the ſame again, *&c.* and this is diſtinguiſhed by daſhing the firſt and laſt figures.

And the like in the 9th example above.

There is ſeldom a neceſſity of obtaining more than 6 places in the decimal or quotient, theſe being ſufficiently exact for moſt uſes..

It may be obſerved, in each of the three laſt Examples, that one or more of the firſt places of the quotient, are poſſeſſed by cyphers; and this is, becauſe two or more cyphers are uſed in the dividend before a digit ariſes on the quotient.

CASE II.

To reduce the different denominations of Money, Weights, Meaſures, &c. *to their equivalent decimal values.*

RULE.

Write the given denominations, or parts, orderly under each other; the inferior or leaſt parts, being uppermoſt: Let theſe be dividends.

Againſt each part, on the left-hand, write the number thereof, contained in one of its next ſuperior: Let theſe be diviſors.

Then, beginning with the upper one, write the quotient of each diviſion, as decimal parts, on the right-hand of the dividend next below it; and let this mixed number be divided by its diviſor, *&c.*

And the laſt quotient will be the decimal ſought.

EXAMPLES.

I. *Reduce* 10 s. 8 d. *to its equivalent decimal of a* pound ſterling.

12 | 8
20 | 10,*6*
   | 0,5*3*.

Therefore 0,5*3* l. *is equal to* 10 s. 8 d.

II. *What decimal of* 1 l. *is equivalent to* 13 s. 10 ½ d. ?

4 | 2
12 | 10,5
20 | 13,875
   | 0,69375 l.

III. *What decimal of* 1 l. *is equal to* 15 s. 9 ¾ d. ?

4 | 3
12 | 9,75
20 | 15,8125
   | 0,790625 l.

IV. *What decimal of* 1 l. *is equal to* 19 s. 11 ¼ d. ?

4 | 1
12 | 11,25
20 | 19,9375
   | 0,996875 l.

V. *What decimal part of a pound* troy, *is equivalent to* 10 *oz.* 18 *dwts.* 16 *grs.* ?

24 { 4 | 16
24 { 6 | (4
20 | 18,*6*
12 | 10,9*3*
   | 0,9*1* ℔ troy.

VI. *What decimal part of a C. wt. is equivalent to* 3 *qrs.* 16 ℔ 12 *oz.* averdupoiſe.

16 { 4 | 12
16 { 4 | (3
28 { 4 | 16,75
28 { 7 | (4, 1875
4 | 3,598214, *&c.*
  | 0,899553 *C.w.*

VII. *What decimal part of a foot, is equal to* 10 *in.* 9 *pts.* 7 *ſec.*

12 | 7
12 | 9,58*3*
12 | 10,7986*1*
   | 0,899884 *f.*

VIII. *What decimal part of a degree of a circle, is equal to* 48 *min.* 37 *ſec.* 54 *thirds.*

60 | 54
60 | 37,9
60 | 48,631*6*
   | 0,81052*7* *deg.*

Note, For ſterling money, the decimal may be wrote in one line, by the following,

RULE.

Write half of the greateſt even number, in the given ſhillings, for the place of primes.

Let the farthings, in the given pence and farthings, poſſeſs the places of the ſeconds and thirds. Obſerving, if the given ſhillings are odd, to increaſe the place of ſeconds by 5.

And to increaſe the thirds by as many units as there are times 24 in the pence and farthings.

Divide half the number of farthings, in the pence and farthings (rejecting 24, or ſixpence, if there is one) by 12, the quotient written after the three places before found, will give the decimal required.

EXAMPLES.

I.	10 *s.* 8 *d.*	is equal to	$0{,}5\cancel{3}$ *l.*
II.	13 *s.* 10 $\frac{1}{2}$ *d.*	——	0,69375 *l.*
III.	15 *s.* 9 $\frac{3}{4}$ *d.*	——	0,790625 *l.*
IV.	19 *s.* 11 $\frac{1}{4}$ *d.*	——	0,996875 *l.*
V.	1 *s.* 10 $\frac{1}{4}$ *d.*	——	$0{,}092708\cancel{3}$ *l.*
VI.	0 *s.* 8 $\frac{3}{4}$ *d.*	——	$0{,}036458\cancel{3}$ *l.*
VII.	0 *s.* 2 $\frac{1}{2}$ *d.*	——	$0{,}01041\cancel{6}$ *l.*
VIII.	0 *s.* 0 $\frac{3}{4}$ *d.*	——	0,003125 *l.*

One of theſe examples explained, will make the rule familiar.

In the V. *viz.* 1 *s.* 10 $\frac{1}{4}$ *d.* half of 1 *s.* is 0, write 0 in the place of primes: 10 $\frac{1}{4}$ *d.* is 41 farthings; and 1 added (for the 24 contained in 41,) makes 42; and 5,0 added (for the odd ſhilling,) makes 92; therefore the three firſt places of the decimal, are 0,092: now 24 taken from 41, leaves

17;

17; its half is 8,5; which divided by 12, gives 7803; theſe wrote, as they ariſe, after the former three places, make 0,0927803 for the decimal required.

CASE III.

III. *A decimal fraction being given; to find its equivalent value, in inferior denominations.*

RULE.

Multiply the given decimal, by the number of parts in the next leſſer denomination; from the proproduct, cut off as many places to the right-hand, as there are in the given decimal.

Multiply theſe by the parts in the next leſſer denomination, and from this product cut off, as before.

And thus proceed until the leaſt denomination is arrived at, then the ſeveral parts cut off on the left-hand, are equivalent to the given decimal.

EXAMPLES.

I. *What is the value of* 0,72896 *l.?*

0,72896
20
14,57920
12
6,95040
4
3,80160

Anſwer, 14 *s.* 6¾ *d.*

II. *What is the value of* 0,92384 *l.?*

0,92384
20
18,47680
12
5,72160
4
2,88640

Anſwer, 18 *s.* 5½ *d.*

III. *What is the value of 0,798645 of C. wt. averdupoiſe?*

0,798645
4 *Quarters.*
3,194580
28 *Pounds.*
1556640
389160
5,448240
16 *Ounces.*
7,171840
Anſwer 3 *qrs.* 5 ℔. 7 *oz.*

IV. *What is the value of 0,87628 of a ℔ troy?*

0,87628
12 *Ounces.*
10,51536
20 *Pennyweights.*
10,30720
24 *Grains.*
122880
61440
7,37280
Anſw. 10*oz.* 10*dwt.* 7*gr.*

But the value of the decimal part of a pound ſterling may be expreſſed in one line; thus.

Double the place of primes for ſhillings, and if the ſecond place be 5, or exceed 5, reckon one ſhilling more: the figures in the ſecond and third places [rejecting 5 in the ſecond place] are ſo many farthings, abating one for every 24.

EXAMPLES.

I. The value of 0,92763*l.* is 18*s.* 6½*d.*
II. - - - - 0,87638*l.* is 17*s.* 6½*d.*
III. - - - 0,09937*l.* is 1*s.* 11¾*d.*
IV. - - - 0,0428 *l.* is 10¼*d.*
V. - - - 0,0095 *l.* is 2¼*d.*

SECTION III.

ADDITION and SUBSTRACTION.

CASE I.

To add or ſubſtract finite Decimals.

RULE.

ADD, or ſubſtract, as in whole numbers, and from the ſum, or difference, cut off as many decimal places as are the greateſt number of decimals in any of the given expreſſions: but obſerve, that the ſeparating commas in each expreſſion, be placed directly underneath each other; for then units, *&c.* (if any) will fall under units, *&c.* and primes, ſeconds, thirds, *&c.* under primes, ſeconds, thirds, *&c.*

EXAMPLES in ADDITION.

347,256	3468,04973	8267,4031
5,61739	24,3675	8,965
0,1725	148,952	453,720853
43,·	37,284695	76,98124
36,5	5,125	0,2738107
432,54589	3683,778925	8807,3440037

EXAMPLES in SUBSTRACTION.

From	384,76215	Minuend.	From	426,8
Take	86,2095.	Subtrahend.	Take	379,604832
	298,55265	Differ. or Rem.		47,195168

CASE

CASE II.

To add decimals, wherein there are ſingle recurring figures.

RULE.

Make every line end at the ſame place, by filling up the vacancies with the recurring digits, and annexing a cypher or cyphers, to the finite terms: then add as before; only increaſe the ſum of the right hand row, with as many units as it contains nines, and the figure in the ſum under that place will be a circulate.

EXAMPLES.

8439,6548	5391,357	217,849$\dot{6}$	876,29$\dot{3}$
281,04$\dot{6}$	72,3$\dot{8}$	42,17$\dot{6}$	5,8764289
7042,3$\dot{5}$	187,2$\dot{1}$	,52$\dot{3}$	,0358$\dot{6}$
9,83	4,296$\dot{5}$	58,30048	628,4593$\dot{8}$

In each of theſe examples there are ſingle recurring figures, which before they are added, muſt be made to end together, and then they will ſtand as follows:

8439,6548$\dot{0}$	5391.457$\dot{0}$	217,849$\dot{6}$66
281,04$\dot{6}$66	72,3$\dot{8}$88	42,17$\dot{6}$666
7042,3$\dot{5}$555	187,2$\dot{1}$11	,52$\dot{3}$333
9,83$\dot{7}$77	4,296$\dot{5}$	58,30048$\dot{0}$
15772,8948$\dot{0}$	5655,253$\dot{5}$	318,85014$\dot{6}$

876,29$\dot{3}$33333
5,8764289$\dot{0}$
,0358$\dot{6}$666
628,4593$\dot{8}$888
1510,6650177$\dot{8}$

Here it may be obſerved, that in each example the circulates are carried one place farther than the finite expreſſions, and to the ſum of that row, there are as many units added, as there were nines in the ſum.

CASE III.

To find the difference between two decimal fractions with ſingle circulates.

RULE.

Make both end together as in addition: and if the right-hand figure of the ſubtrahend (being a circulate) be bigger than the figure over it in the minuend, inſtead of borrowing 10, as in ſubſtraction of whole numbers or finites; borrow 9 in this place, the reſt as uſual, and the right-hand place of the remainder will be a circulate.

From	$476,3\dot{2}$	$289,576$	$325,791\dot{8}$	$643,9207$
Take	$84,769\dot{7}$	$92,584\dot{6}$	$37,095$	$583,7\dot{6}$

These examples being made to end together, as before directed, will ſtand thus:

$476,3\dot{2}22$	$289,5760$	$325,791\dot{8}$	$643,92070$
$84,769\dot{7}$	$92,584\dot{6}$	$37,0950$	$583,7\dot{6}666$
$391,552\dot{4}$	$196,991\dot{3}$	$288,696\dot{8}$	$60,1540\dot{3}$

SEC-

SECTION IV.

MULTIPLICATION.

CASE I.

When both factors are finite decimals.

RULE.

PLACE the factors, and multiply them as in whole numbers; and from the product, towards the right-hand, cut off as many places for decimals, as there are fractional parts in both factors together.

But if it ſo happen, that there are not ſo many places in the product, ſupply the defect with cyphers to the leſt-hand.

3684,7928	,2365	,0347
84,216	,2435	,0236
221087568	11825	2082
36847928	7095	1041
73695856	9460	694
147391712	4730	,00081892
294783424	,05758775	
310318,5104448		

In the firſt example, there being four decimal places in the multiplicand, and three in the multiplier, which together are ſeven; therefore cut off ſeven figures from the right-hand of the product for decimals; thoſe to the left-hand being integers.

In each of the ſecond and third examples, the decimal places in both factors are eight, but there ariſes in the ſecond example, only ſeven figures in the product, and in the third, only five; therefore in the ſecond example, annex one cypher, and in the third, three cyphers, to ſupply the defect.

CASE II.

If the right-hand figure of the multiplicand be a circulate.

RULE.

Multiply the multiplicand as before, by every figure in the multiplier; obſerving to increaſe the right-hand figure of each reſulting line, by as many units as there are nines in the firſt product of that line; and the right-hand figure of each line will be a circulate; therefore in the adding the ſeveral lines together, make them end at the ſame place, as ſhewn in addition.

EXAMPLES.

Firſt.

$$\begin{array}{r} 821{,}734\dot{5} \\ {,}8 \\ \hline 657{,}3876\dot{4} \\ \hline \end{array}$$

Second.

$$\begin{array}{r} 3796{,}27\dot{3} \\ 4{,}6 \\ \hline 2277764\dot{0} \\ 1518509\dot{3}3 \\ \hline 17462{,}857\dot{3} \\ \hline \end{array}$$

Third.

$$\begin{array}{r} 8946{,}83706\dot{6} \\ 48{,}57 \\ \hline 626278594\dot{6} \\ 447341853\dot{3}3 \\ 715746965\dot{3}33 \\ 357873482\dot{6}666 \\ \hline 434547{,}876328\dot{0} \\ \hline \end{array}$$

CASE

CASE III.

When the multiplier is a ſingle circulate.

RULE.

Multiply by it, as tho' it was a finite digit, ſetting the product one place forwarder than ordinary, towards the left hand; divide the reſult by nine, continuing the quotient (if needful) till it arrives at a circulate; then beginning at the place under the right-hand figure of the multiplicand, cut off for fractions as before, and this will be the true Product.

EXAMPLES.

Firſt.

438,6297
$,\dot{6}$
9)263177782
292,4198$\dot{0}$

Second.

5820,39462
$,3\dot{7}$
9)4074276234
452697359$\dot{3}$
1746118386
2198,815745$\dot{3}$

Third.

47,63
2,84$\dot{5}$
9) 23815
2646$\dot{1}$
19052
38104
9526
135,5338$\dot{1}$

2,84$\dot{5}$
47,63
853$\dot{6}$
1707$\dot{3}$3
1991$\dot{8}$88
1138$\dot{2}$222
135,5338$\dot{1}$

This third Example is wrought by Caſe II. and III. and the reſults are exactly the ſame.

In this caſe, if there are any other figures in the multiplier, beſide the circulate, multiply by them like finite digits.

CASE IV.

When the multiplicand and multiplier are each a ſingle circulate.

RULE.

In multiplying the multiplicand by each figure in the multiplier, obſerve the directions given in Caſe II. but the firſt line (or that line produced by multiplying the multiplicand, by the circulate in multiplier) muſt be managed as directed in Caſe III.

EXAMPLES.

463,9704̸
8,642̸
9)9279408̸
1031045̸432, &c.
1855881777̸, &c.
27838226̸66, &c.
37117635̸5555, &c.
4009,735685̸43, &c.

862357,97̸
36,16̸
9)517414753̸
57490528̸148
86235797̸
431178961̸1̸
258707376̸66
30326253,598̸14̸

0,5392765̸
935,7̸
9) 37749358̸
4194373̸2098, &c.
16178296̸
26963827̸7
48534890
514,3499947̸65432098̸

By dividing by nine in each of theſe examples there reſults a compound circulate; now having finiſhed the multiplication, make all the lines end together, but no farther than the firſt figure of the compound circulate; then having added the ſeveral lines together as before directed, daſh the right-hand figure of the product, for the firſt figure of a compound circulate; the reſt of which ſet orderly after, as they follow in the given compound circulate.

CASE V.

Two decimal factors being given, to reſerve in their product any number of places.

RULE.

Under that place in the multiplicand, thought neceſſary to be retained in the product, write the units place of the multiplier, and invert the order of all its other places; that is, write the decimals on the left, and the integers (if any) on the right.

In multiplying, omit thoſe places in the multiplicand, to the right of the digit multiplying by, and let the right-hand place of every line, ſtand under each other.

In each line, let the loweſt place be increaſed by the carriage, which would ariſe from the omitted places; that is, carrying 1 from 5 to 15; 2 from 15 to 25; 3 from 25 to 35, *&c.* inſtead of carrying 1 for every 10, *&c.*; and the Sum of theſe lines will give the product generally exact.

This Rule is of uſe, to contract the work that would ariſe in multiplying with many decimal places, by omitting the ſuperfluous ones.

EXAMPLES.

Multiply 384,672158 by 36,8345.

Now ſeeing there would be 10 decimal places in the product, whereof the greateſt Part are unneceſſary; therefore keep only four decimal places in the product.

384,672158	Muliplicand.	384,672158	
5438,63	Multiplier inverted.	36,8345	
115401647 .		1923	360790
23080329 ..		15386	88632
3077377 ...		115401	6474
115402		3077377	264
15387		23080329	48
1923		115401647	4
14169,2065		14169,2066	038510

Here the Example is wrought both ways, by which may be eaſily ſeen what is ſaved by the laſt Rule.

In this Example, becauſe it is intended to keep 4 decimal places in the product, ſet 6, the unit's place of the multiplier under 1, the *4th* place in decimals of the multiplicand, and invert the order of all the reſt of the figures: Then ſay three times 8 is 24, and carry 2; 3 times 5 is 15, and 2 is 17, now ſet down the 7 and carry 1, *&c.* becauſe this is the product ariſing by multiplying the 5 that ſtands over the 3.

Again, 6 times 8 is 48, and carry 5; 6 times 5 is 30, and 5 is 35, and carry 3; 6 times 1 is 6, and 3 is 9. Now being come to the figure over the 6, ſet down 9, *&c.*

Again, 8 times 5 is 40, and carry 4; 8 times 1 is 8, and 4 is 12, and carry 1; 8 times 2 is 16, and 1 is 17; now being come to the figure over the 8, ſet down 7, and carry 1, *&c.* Proceeding in like manner with every figure in the inverted multiplier, till all is done.

Multiply 3,141592 by 52,7438, and reſerve 4 decimal places in the product.

```
 3,141593
 8347,25
 ————————
 1570796
   62832
   21991
    1257
      94
      25
 ————————
 165,6995
 ————————
```

Multiply 257,356 by 76,48, and to have the product only in whole numbers.

```
 257,356            257,356
 84,67                76,48
 ———————        ————————————
 18015               20|58848
   1544             102|9424
    103            1544|136
     20           18014|92
 ———————        ————————————
  19682         19682,58688
 ———————        ————————————
```

SEC-

SECTION V.

DIVISION.

CASE I.

When the diviſor and dividend are finite decimals.

RULE.

DIVIDE as in integers, and from the right-hand of the quotient, point off for decimals, ſo many places, as the decimal places uſed in the dividend, exceed thoſe of the diviſor; and thoſe to the left, if any, are integers.

If the places in the quotient, are not as many as this Rule requires, ſupply the defect with cyphers, on the left-hand.

But if the decimal places in the diviſor, be more than thoſe in the dividend, add cyphers as decimals to the dividend, till the number of decimal places in the dividend, is, at leaſt, equal to thoſe in the diviſor, and the quotient will be integers until all theſes cyphers are uſed.

```
43,6) 3424,6056 (78,546
        3726
         2380
          2005
           2616
```

0,675) 3877875,000 (5745000
5028
3037
3375
000

0,347) ,0081892 (,0236
1249
2082

If theſe Examples be compared with the foregoing Rule, it will be very eaſy to ſee how they are performed, and the quotient rightly adjuſted.

Or, the place of the firſt digit in the quotient, will always be equal to that place of the dividend, under which, falls the units of the diviſor, when multiplied by that quotient digit.

Thus in Ex. I. the diviſor 43,6, multiplied by 7, the firſt digit in the quotient; the place of units in the product, will fall under the place of *tens* in the dividend; therefore the place of the firſt digit, 7, in the quotient, will be that of *tens*.

In Ex. II. the place of units in the diviſor, falls under the place of *millions* in the dividend; therefore 5, the firſt digit in the quotient, will be in the place of *millions*.

In Ex. III. the place of units in the diviſor, falls under that of ſeconds in the dividend; therefore 2, the firſt digit in the quotient, will poſſeſs the place of ſeconds; and conſequently a cypher will be in the place of primes.

CASE II.

If the dividend be a circulate.

RULE.

To the remainders, bring down the recurring figure, until the quotient is as exact as required.

```
4,72) 8,3̇3 (1,765536, &c.
       ----
       3613
       ----
        3093
        ----
         2613
         ----
          2533
          ----
           1733
           ----
            317, &c.

54,283)0,876527̇ (,016147, &c.
        -------
        333697
         ------
          79997
          ------
          257147
           ------
           400157
           ------
            20176, &c.
```

CASE III.

If the diviſor be a recurring decimal.

RULE.

Write the diviſor and dividend in the order of diviſion; and under theſe, write them a ſecond time; but each remov'd as many places to the right, as are the number of circulating places in the diviſor; then ſubſtracting the lower line from the upper one, let the remainder be the diviſor and dividend; and the quotient of theſe will be the true one.

EXAMPLES.

$$\begin{array}{lrrl}
\textit{First.} & 8946{,}8370\cancel{6}) & 434547{,}876328 &)48{,}57 \\
 & \underline{894{,}68370} & \underline{43454{,}7876328} & \\
 & 8052{,}15336) & \underline{391093{,}0886952} & \\
 & & \underline{6900695429} & \\
 & & \underline{4589727415} & \\
 & & \underline{5636507352} & \\
 & & 0000000000 &
\end{array}$$

It will be eaſy to ſee how this example is performed, by obſerving the rule; and by comparing it with Example III to Caſe I. in multiplication, may be ſeen how theſe two rules prove each other.

$$\begin{array}{lrrl}
\textit{Second.} & 748{,}6\cancel{4}) & 47{,}46405\cancel{7} & (,0634 \\
 & \underline{74{,}86} & \underline{4{,}746405} & \\
 & 673{,}78) & \underline{42{,}717652} & \\
 & & \underline{229085} & \\
 & & \underline{269512} & \\
 & & \underline{0} &
\end{array}$$

$$\begin{array}{lrrl}
\textit{Third.} & 2{,}84\cancel{5}) & 135{,}5338\cancel{1} & (47{,}63 \\
 & \underline{284} & \underline{13{,}55338} & \\
 & 2{,}561) & \underline{121{,}98043} & \\
 & & \underline{19540} & \\
 & & \underline{16134} & \\
 & & \underline{7683} & \\
 & & \underline{0} &
\end{array}$$

Fourth.

$$\begin{array}{llll}
\textit{Fourth} & 862357{,}9\cancel{7}) & 30326253{,}59\cancel{8}1\cancel{4} & \textit{ſee Ex. p. } 18. \\
 & \underline{86235{,}79} & \underline{3032625{,}35981} & \\
 & 776122{,}13 & \underline{27293628{,}238\cancel{3}3} & (35{,}1\cancel{0},\ \&c. \\
 & & \underline{400996433} & \\
 & & \underline{129353688} & \\
 & & \underline{51741475}3 & \\
 & & \underline{51741475,}\ \&c. & \\
\end{array}$$

$$\begin{array}{llll}
\textit{Fifth.} & 76{,}4\cancel{7}) & 8293{,}764\cancel{8} & (108{,}44 \\
 & \underline{7{,}64} & \underline{829{,}3764} & \\
 & 68{,}83) & \underline{7464{,}3879} & \\
 & & \underline{58138} & \\
 & & \underline{30747} & \\
 & & \underline{32159} & \\
 & & \underline{4627} & \\
\end{array}$$

CASE IV.

To contract the work of diviſion, when the diviſor conſiſts of many decimal places.

RULE.

Let each remainder be a new dividend, and for each ſuch new dividend, point off one figure from the right-hand of the diviſor; obſerving at each multiplication to have regard to the increaſe of the figures ſo cut off, as in contracted multiplication.

EXAMPLES.

```
384,672158) 14169,2066239510 (36,8345
   .....    11540,16474
            ----------
              262904188 .
              230803295 .
              ---------
               32100893 . .
               30773772 . .
               --------
                1327121 . . .
                1154016 . . .
                -------
                 173105 . . . .
                 153869 . . . .
                 ------
                  19236 . . . . .
                  19234 . . . . .
                  -----

  9,365407) 87,076326 (9,297655
   ......   84,288663
            ---------
             2 787663 .
             1 873081 .
             --------
               914582 . .
               842886 . .
               ------
                71696 . . .
                65558 . . .
                -----
                 6138 . . . .
                 5619 . . . .
                 ----
                  519 . . . . .
                  468 . . . . .
                  ---
                   51 . . . . .
                   47 . . . . . .
                   --
                    4 . . . . . .
```

This will not be difficult if it be carefully examined.

SECTION VI.

A decimal fraction being given, to find its leaſt equivalent vulgar one.

CASE I.

If the decimal be finite.

RULE.

UNDER the given decimal, write its proper denominator; then the terms of this fraction, divided by their greateſt common meaſure, will give the leaſt equivalent vulgar fraction required.

Note, The greateſt common meaſure of two given numbers, is found by dividing the greater by the leſſer, the leſſer by the remainder, *&c.* always dividing the laſt diviſor by the laſt remainder, until nothing remains; and the laſt diviſor is the greateſt common meaſure.

EXAMPLES.

I. Required the leaſt vulgar fraction equivalent to the decimal 0,5?

$$\text{Then } 0{,}5 \overset{*}{=} \frac{5}{10} = \frac{1}{2}.$$

II. What is the leaſt vulgar fraction to 0,75?

$$\text{Now } 0{,}75 = \frac{75}{100} = \frac{3}{4}.$$

III. What is the leaſt vulgar fraction to 0,6?

$$\text{Now } 0{,}6 = \frac{6}{10} = \frac{3}{5}.$$

---

* For the explanation of the ſigns, ſee Sect. XI.

IV. What is the leaſt vulgar fraction to 0,625?

Now $0{,}625 = \frac{625}{1000} = \frac{5}{8}$.

For 625)1000(1
375)625(1
250)375(1
125)250(2
0

And $125)\frac{625}{1000}(\frac{5}{8}$.

V. What is the leaſt vulgar fraction to 0,5625?

Now $0{,}5625 = \frac{5625}{10000} = \frac{9}{16}$.

For 5625)10000(1
4375)5625(1
1250)4375(3
625)1250(3
0

And $625)\frac{5625}{10000}(\frac{9}{16}$.

CASE II.

If the given decimal be a recurring one.

RULE.

Make the given decimal, the numerator of a vulgar fraction, whoſe denominator ſhall conſiſt of as many nines, as there are recurring places in the given decimal; the terms of this fraction, divided by their greateſt common meaſure, will give the leaſt equivalent vulgar fraction required.

If one or more of the left-hand places in the given decimal be cyphers, annex as many cyphers to the right-hand of the nines in the denominator.

EXAMPLES.

I. Required the leaſt equivalent vulgar fraction, to $0,\dot{3}$?

Now $0,\dot{3} = \frac{3}{9} = \frac{1}{3}$.

II. What is the leaſt vulgar fraction to $0,\dot{2}5\dot{7}$?

Now $0,\dot{2}5\dot{7} = \frac{257}{999}$.

III. What is the leaſt vulgar fraction to $0,0\dot{5}9440\dot{5}$?

Now $0,0\dot{5}9440\dot{5} = \frac{594405}{9999990} = \frac{17}{286}$.

For the greateſt common meaſure to $\frac{594405}{9999990}$ is 34965.

And 34965) $\frac{594405}{9999990}$ ($\frac{17}{286}$

(See Exam. VII. P. 6.)

CASE III.

When part of the given decimal is finite, and part circulates.

RULE.

1. To the right-hand of as many nines, as there are recurring places, annex as many cyphers as there are finite places; and let this be the denominator.
2. Multiply the nines in the denominator, by the finite part; to the product, add the recurring part, and make this the numerator.

3. The terms of this vulgar fraction divided by their greateſt common meaſure, will give the leaſt equivalent fraction required.

Note, The firſt and ſecond Caſes, are included in this.

EXAMPLES.

I. Required the leaſt equivalent vulgar fraction to the decimal $0,5\acute{3}$?

Now $0,5\acute{3} = \frac{48}{90} = \frac{8}{15}$.

For 90 is the denominator,

And $\overline{9 \times 5} + 3 = 48$ is the numerator.

II. What is the leaſt vulgar fraction to $0,58\acute{3}$?

Now $0,58\acute{3} = \frac{525}{900} = \frac{7}{12}$.

For $0,58\acute{3} = \frac{58}{100} + \frac{3}{900}$; And $\overline{58 \times 9} + 3 = 525$.

And the greateſt common meaſure is 75.

Then $75\,\Big)\frac{525}{900}\Big(\frac{7}{12}$.

III. What is the leaſt vul. fraction to $0,008\acute{4}9713\acute{3}$?

Now $0,008\acute{4}9713\acute{3} = \frac{8497125}{999999000} = \frac{83}{9763}$.

For $0,008\acute{4}9713\acute{3} = \frac{8}{1000} + \frac{497133}{999999000}$.

And $\overline{8 \times 999999} + 497133 = 8497125$.

And the greateſt common meaſure is 102375.

Then $102375\,\Big)\frac{8497125}{999999000}\Big(\frac{83}{9763}$.

See Example IX. P. 6.

The three Caſes may be ſolved by the following,

RULE.

1. Under the given decimal ſet its proper denominator.
2. Write this vulgar fraction under itsſelf; but remov'd as many places forward to the right-hand, as there are recurring places.
3. Subtract the under numerator from the upper one, and the under denominator from the upper one; then will the remainders conſtitute a vulgar fraction equivalent to the given decimal.

Reduce this to its leaſt terms as before.

EXAMPLES.

I. To find a vulgar fraction to $0{,}58\dot{3}$.

$$\text{From } \frac{583}{1000}$$

$$\text{Take } \frac{58}{100}$$

$$\text{Remains } \frac{525}{900} = 0{,}58\dot{3}\text{; as before.}$$

II. Required the vulgar fraction to $0{,}008\dot{4}9713\dot{3}$.

$$\text{From } \frac{008497133}{1000000000}$$

$$\text{Take } \frac{008}{1000}$$

$$\text{Leaves } \frac{8497125}{999999000} = 0{,}008\dot{4}9713\dot{3}\text{ as before.}$$

SCHOLIUM.

The truth of theſe Rules, and indeed of all the Rules concerning recurring decimals, may be eaſily examined, if it be conſidered, that every circulating or recurring decimal, is a geometrical ſeries infinitely decreaſing to 0.

Thus $0,\acute{3}$, *&c.* $= \frac{3}{10} + \frac{3}{100} + \frac{3}{1000}$, *&c.*

And

$0,\acute{2}5\acute{7}$, *&c.* $= \frac{257}{1000} + \frac{257}{1000000} + \frac{257}{1000000000}$, *&c.*

And the ſum of ſuch a decreaſing ſeries, is equal to the ſquare of the firſt term, divided by the difference between the firſt and ſecond terms.

SECTION VII.

Of TABLES.

IN decimal computations, 'tis of uſe to have Tables of the decimal values of the parts of coin, weight, meaſures and time; therefore the following Tables, and the manner of conſtructing them, are here introduced.

CONSTRUCTION *of* TABLE I.

I. The ſhillings 19, 18, 17, 16, *&c.* are ſeparately divided by 20, and the ſeveral quotients are the decimals of their reſpective ſhillings.

II. The decimals of 11, 10, 9, *&c.* ſhillings, are divided by 12, and the quotients are the decimals of the pence 11, 10, 9, *&c.*

III. The decimals of 3, 2, 1, pence, are divided by 4, and the quotients are the decimals of the farthings 3, 2, 1.

CONSTRUCTION *of* TABLL II.

I. The ounces 11, 10, 9, 8, *&c.* are divided by 12, the ounces in a troy pound, and the quotients, are the decimals of thoſe ounces.

II. The decimal of one ounce, is divided by 20, (the penny weights in one ounce) and the quote is the decimal of 1 penny weight; which multiplied by the other penny weights, gives their reſpective decimals. And from theſe are the decimals of grains, conſtructed in the ſame manner.

CONSTRUCTION *of* TABLE III.

I. Becauſe 20 hundred is one ton, therefore the decimals of the ſhillings will ſerve for the hundred weights, ſuppoſing 1 ton the integer.

II. The decimal of 1 ſhilling (*i e.* 1 hundred weight) is divided by 4 (the quarters in one hundred weight) and the quote, is the decimal of 1 quarter, from whence the decimals of the other quarters are obtained, as are alſo the decimals of pounds.

HUNDRED WEIGHT *the* INTEGER.

The decimals of the quarters are ,25, ,5 and ,75; and dividing the decimal of 1 quarter by 28, (the Pounds in a quarter) gives the decimal of 1 pound; from whence the decimals of the other pounds are obtained; as are alſo the decimals of ounces; and from them the decimals of drams.

ONE POUND *the* INTEGER.

One ounce is divided by 16, (the ounces in 1 pound) and it gives the decimal of 1 ounce; from whence the decimals of the other ounces are obtained; as are alſo the decimals of drams.

After the ſame manner are the other Tables of meaſures and time conſtructed; having always a due regard how many of a leſſer denomination are contained in a ſuperior one.

As theſe Tables are only carried to ſix places (excepting in ſome particulars) whenever the ſeventh figure would have been more than a 5, the ſixth place has been increaſed with unity; otherwiſe, the ſixth place is given as it ariſes in the work.

The uſe of theſe Tables are obvious; but to prevent all doubts obſerve the following,

EXAMPLES.

I. *What is the decimal value of* 7 *oz.* 16 *dwt.* 18 *grs.*?

In TABLE II.

againſt 7 *oz.* is ,5833333
againſt 16 *dwt.* is ,0666666
againſt 18 *grs.* is ,0031250

Theſe added, their ſum is 0,653125, the decimal required.

II. *What is the decimal value of* 46 *gallons and* 5 *pints?*

In TABLE IV.

againſt 40 gallons is ,634920
againſt 6 gallons is ,095238
againſt 5 pints is ,009921

Theſe added, their Sum is 0,740079, the decimal required.

After the ſame manner, are decimals of other denominations collected.

N. B. If the number of parts wanted are not found in the Tables, take it out at twice; thus: For 17 *dwt.* add 10 and 7 together; and the like for any other.

T A-

TABLE I.

Coin, 1 ℔ the Integer.

S.	Dec.	S.	Dec.
19	,95	9	,45
18	,9	8	,4
17	,85	7	,35
16	,8	6	,3
15	,75	5	,25
14	,7	4	,2
13	,65	3	,15
12	,6	2	,1
11	,55	1	,05
10	,5		

Pence.	*Decimals.*
11	,04583̸ .
10	,0416̸ . .
9	,0375 . .
8	,03̸ . . . .
7	,02916̸ .
6	,025 . . .
5	,02083̸ .
4	,016̸ . . .
3	,0125 . .
2	,0083̸ . .
1	,00416̸ .

Farthings.	*Decimals.*
3	,003125
2	,0020833̸
1	,0010416̸

Note. *The following Table of Ounces will serve for Inches and any thing where 12 is the Integer.*

TABLE II.

Troy Weight.

1 ℔ the Integer.

Ounces.	*Decimals.*
11	,916 . . .
10	,83̸ . . . .
9	,75 . . . .
8	,6̸ . . . . .
7	,583̸ . . .
6	,5 . . . . .
5	,416̸ . . .
4	,3̸ . . . . .
3	,25 . . . .
2	,16̸ . . . .
1	,083̸ . . .

Dwts.	*Decimals.*
10	,0416̸ . .
9	,0375 . .
8	,03̸ . . . .
7	,02916̸ .
6	,025 . . .
5	,02083̸ .
4	,016̸ . . .
3	,0125 . .
2	,0083̸ . .
1	,00416̸ .

Gs.	
10	,001736
9	,001563
8	,001389
7	,001215
6	,001042
5	,000868
4	,000694
3	,000521
2	,000347
1	,000173

TABLE III.

Avoird. 1 Ton the Inte.

Qrs. Cws.	*Decimals.*
3	,0375 . .
2	,025 . . .
1	,0125 . .

Pounds.	*Decimals.*
20	,008928
10	,004464
9	,004018
8	,003571
7	,003125
6	,002678
5	,002232
4	,001785
3	,001339
2	,000892
1	,000446

1 Pound the Integer.

Ounces.	*Decimals.*
10	,625 . . .
9	,5625 . .
8	,5 . . . . .
7	,4375 . .
6	,375 . . .
5	,3125 . .
4	,25 . . . .
3	,1875 . .
2	,125 . . .
1	,0625 . .

TABLE III.

Avoird. 1 lb the Integ.

Drams.	*Decimals.*
10	,039062
9	,035156
8	,031250
7	,027344
6	,023437
5	,019531
4	,015625
3	,011718
2	,007812
1	,003906

1 Cwt. the Integer.

Pounds.	*Decimals.*
20	,178571
10	,089285
9	,080357
8	,071428
7	,0625
6	,053571
5	,044642
4	,035714
3	,026785
2	,017857
1	,008928

Ounces.	*Decimals.*
10	,005580
9	,005022
8	,004464
7	,003906
6	,003348
5	,002790
4	,002232
3	,001674
2	,001116
1	,000558

TABLE III

Avoirdupoiſe.

1 *Cwt. the Integer.*

Drams.	*Decimals.*
10	,000349
9	,000314
8	,000279
7	,000244
6	,000209
5	,000174
4	,000139
3	,000104
2	,000069
1	,000034

TABLE IV.

Liquid Meaſure.

1 *Tun the Integer.*

Gallons.	*Decimals.*
200	,793651
100	,396825
90	,357141
80	,317460
70	,27 . . . .
60	,238095
50	,198412
40	,158730
30	,119047
20	,079365
10	,039682
9	,035714
8	,031746
7	,027 . . .
6	,023809
5	,019841
4	,015873
3	,011904
2	,007936
1	,003968

TABLE V.

Liquid Meaſure.

1 *Tun the Integer.*

Pints.	*Decimals.*
7	,003472
6	,002976
5	,002480
4	,001984
3	,001488
2	,000992
1	,000496

1 *Hogſhead the Intege.*

Gallons.	*Decimals.*
60	,952381
50	,793651
40	,634921
30	,476190
20	,317460
10	,158730
9	,142857
8	,126984
7	,1 . . . . .
6	,095238
5	,079365
4	,063492
3	,047619
2	,031746
1	,015873

Pints.	*Decimals.*
7	,013889
6	,011905
5	,009921
4	,007937
3	,005952
2	,003968
1	,001984

TABLE V.
Long Meaſure.
1 *Mile the Integer.*

Yards.	*Decimals.*
1000	,568182
900	,511364
800	,454545
700	,397727
600	,340909
500	,284091
400	,227272
300	,170454
200	,113636
100	,056818
90	,051136
80	,045454
70	,039773
60	,034091
50	,028409
40	,022727
30	,017045
20	,011364
10	,005682
9	,005114
8	,004545
7	,003977
6	,003409
5	,002841
4	,002273
3	,001704
2	,001136
1	,000568

TABLE V.
Long Meaſure.
1 *Mile the Integer.*

Feet.	*Decimals.*
2	,0003787
1	,0001894

Inches.	*Decimals.*
9	,0001421
6	,0000947
3	,0000474
1	,0000158

1 *Yard the Integer.*

2	$\cancel{6}$ . . . . .
1	$\cancel{3}$ . . . . .

Inches.	*Decimals.*
11	,30$\cancel{5}$ . . .
10	,2$\cancel{7}$ . . . .
9	,24$\cancel{9}$ . . .
8	,$\cancel{2}$ . . . . .
7	,19$\cancel{4}$ . . .
6	,1$\cancel{6}$ . . . .
5	,13$\cancel{8}$ . . .
4	,$\cancel{1}$ . . . . .
3	,08$\cancel{3}$ . . .
2	,0$\cancel{5}$ . . . .
1	,02$\cancel{7}$ . . .

Qrs. Inch.	*Decimals.*
3	,0208$\cancel{3}$.
2	,013$\cancel{8}$ . .
1	,0069$\cancel{4}$.

TABLE VI.

Time.

1 *Year the Integer.*

Days.	*Decimals.*
300	,821918
200	,547945
100	,273973
90	,246575
80	,219178
70	,191781
60	,164383
50	,136986
40	,109589
30	,082192
20	,054794
10	,027397
9	,024657
8	,021918
7	,019178
6	,016438
5	,013698
4	,010959
3	,008219
2	,005479
1	,002740

TABLE VI.

Time.

1 *Year the Integer.*

Hours.	*Decimals.*
20	,002282
10	,001141
9	,001027
8	,000913
7	,000799
6	,000685
5	,000571
4	,000456
3	,000342
2	,000228
1	,000114
Minutes.	*Decimals.*
50	,0000951
40	,0000761
30	,00005706
20	,00003804
10	,00001902
9	,00001711
8	,00001521
7	,00001331
6	,00001141
5	,00000951
4	,00000761
3	,00000571
2	,00000380
1	,00000190
Qrs. Min.	*Decimals.*
3	,00000142
2	,00000095
1	,00000047

TABLE VI.

Time.

1 *Day the Integer.*

Hours.	*Decimals.*
20	,8$\cancel{3}$ . . . .
10	,41$\cancel{6}$ . . .
9	,375 . . .
8	,$\cancel{3}$ . . . . .
7	,291$\cancel{6}$ . .
6	,25 . . . .
5	,208$\cancel{3}$ . .
4	,1$\cancel{6}$ . . . .
3	,125 . . .
2	,08$\cancel{3}$ . . .
1	,041$\cancel{6}$ . .

Minutes.	*Decimals.*
50	,0347$\cancel{2}$
40	,02$\cancel{7}$ . . .
30	,0208$\cancel{3}$.
20	,013$\cancel{8}$ . .
10	,0069$\cancel{4}$.
9	,00625 .
8	,00$\cancel{5}$ . .
7	,004861
6	,0041$\cancel{6}$
5	,00347$\cancel{2}$
4	002$\cancel{7}$ . .
3	,00208$\cancel{3}$
2	,0013$\cancel{8}$.
1	,00069$\cancel{4}$

Seconds.	*Decimals.*
45	,0005208
30	,0003472
15	,0001736

TABLE VII.

Various Meaſures.

1ſt. *Cloth Meaſures.*

1 *Yard the Integer.*

Qrs.Yard.	*Decimals.*
3	,75
2	,5 .
1	,25

Nails.	*Decimals.*
3	,1875
2	,125 .
1	,0625

Meaſure.

Liquid. | *Dry.*

Integer.

1 *Gall.* | 1 *Quarter.*

Pints.	*Decim.*	*Buſhel.*
7	,875	7
6	,76 . .	6
5	,625	5
4	,5 . .	4
3	,375	3
2	,25 .	2
1	,125	1

Qs.Ps.	*Decim.*	*Pecks.*
3	,09375	3
2	,0625 .	2
1	,03125	1

Decimals.	*Qrs. Pecks.*
,023437	3
,015625	2
,007812	1

Decimals.	*Pints.*
,005859	3
,003906	2
,001953	1

SECTION VIII.

Of Compariſon or Proportion.

I. THE comparing of things, of a like kind to one another, may be conſidered two ways:

Firſt, By how much one thing exceeds, or is greater than another, and this is called *difference.*

Secondly, What part or parts one thing is of another, and this is called *ratio.* And two or more ratio's make a *proportion*, viz.

When one number is to be divided by another, then the ratio of the diviſor to the dividend, is the ſame as the ratio of unity to the quotient.

Or, the diviſor is ſaid to be in the ſame proportion to the dividend; as unity, is in proportion to the quotient; that is, the diviſor is as often contain'd in the dividend, as unity is contain'd in the quotient.

Therefore the ratio of one number to another, is nothing more, than how often that number does contain, or is contain'd in, the other; or it is meaſured by the quotient ariſing from the diviſion of one number by the other.

II. There cannot be leſs than two numbers or terms in any ratio; the firſt of which, or the term by which the compariſon is made, is called the *antecedent*: and the ſecond, or the term to which the firſt is compared, is called the *conſequent*.

III. When two ratios (*i. e.* quotients) are equal, the numbers, or terms, to which theſe ratios belong, are ſaid to be geometrically proportional.

Thus, the ratio of 3 to 12, is the ſame as the ratio of 4 to 16; therefore the numbers 3, 12, 4, 16, are ſaid to be proportionals.

IV. When of ſeveral terms, or numbers, the quotient of the firſt and ſecond is the ſame with that of the 2d and 3d, and the ſame with that of the third and fourth, *&c.* thoſe numbers are ſaid to be in continued geometric proportion: Thus, 4, 12, 36, 108, *&c.* are numbers in continued geometric proportion, for the quotient of any two adjacent terms is 3.

V. If there are three terms in geometrical proportion, the firſt term multiplied by the third, gives a product equal to the ſecond term multiplied by itſelf.

Hence the ſecond term is called *a mean proportional between the extreams:* viz. *between the firſt and third terms.*

VI. If there are four terms in geometrical proportion, the product of the two extream terms is equal to the product of the two mean terms.

Hence the ſecond and third terms are called *mean proportionals between the firſt and fourth terms.*

Now it is evident, that if the product of the ſecond and third terms be divided by the firſt term, the quotient will be the fourth term.

And hence ariſes the method of operating the rule of three.

VII. The rule of three, is ſo called, becauſe three numbers or terms are given to find a fourth proportional: thus, if the numbers 3, 12, and 18, were given to find a fourth proportional.

Multiply the ſecond term by the third, or (which is all one) the third by the ſecond, *viz.* 18 by 12, the Product will be 216; this 216 divided by the firſt term 3, gives 72 in the quotient, for the fourth term.

Now by comparing of the terms together, it will be found, that the ſecond term 12, as often contains the firſt term 3; as the fourth term 72, contains the third term 18: alſo that the third term 18, as often contains the firſt term 3, as the fourth term 72 contains the ſecond term 12; that is, the ratio or proportion of 3 to 12, is the ſame as the ratio of 18 to 72. Alſo the ratio of 3 to 18, is the ſame as the ratio of 12 to 72; and the like in other numbers.

VIII. The rule of three being moſtly concerned in finding a fourth number proportional to three numbers or terms given, differing in ſignification and denomination, the greateſt difficulty lies in ſtating the terms; that is, in placing them in proper order to be multiplied and divided according to the foregoing directions; to do this proceed as follows.

Place that for the ſecond, which has the ſame name with the fourth, or term ſought.

Then conſider, from the nature of the queſtion, whether that fourth term ſhould be leſs, or greater than the ſecond.

If the ſecond ſhould be $\left\{\begin{matrix}\text{leſs}\\\text{greater}\end{matrix}\right\}$ than the 4th, let the $\left\{\begin{matrix}\text{leaſt}\\\text{greateſt}\end{matrix}\right\}$ of the remaining two terms be placed firſt; and the other, for the third term; then the three given terms are truly ſtated.

If any, or all, of the terms be of different denominations, reduce the leſſer ones to the decimal part or parts of the greater, as ſhewn in Sect. II. obſerving, that the firſt and third terms, be reduced to the decimal part or parts of one and the ſame denomination.

Multiply the ſecond and third terms together, by the rules in Sect. IV. divide the product by the firſt term, adjuſt the quotient by the rules given in Sect. V. and this will be the fourth term, or the number ſought, and is to be valued by the rules given in Sect. II.

QUESTION I.

What will 326¼ ℔ of tobacco come to, at 3*s.* 6*d.* for 1½ ℔?

Here the quality of the fourth term is money, *viz.* the worth of 326¼ ℔ of tobacco; and among the three terms given, one is money, *viz.* 3*s.* 6*d.* the worth of 1½ ℔.

Now it is eaſy to ſee, that 326¼ ℔ will come to more than 1½ ℔; therefore the fourth term will be greater than the ſecond; and the terms ſtated, will ſtand thus: if 1½ ℔ coſt 3*s.* 6*d.* what will 326¼ ℔ come to; and the terms reduced into decimals will ſtand thus:

```
If 1,5 ℔ — 0,175 l. — 326,25 ℔
                        0,175
                       ------
                       163125
                     554625
                     --------
          1,5)  57,09375 (38,0625 l.
                120
                ---
                  93
                  ---
                   37
                   ---
                    75
```

Here the third term being multiplied by the ſecond, and the product being divided by the firſt term, gives 38,0625 *l.* in the quotient; which being valued as directed in Sect. II. gives 38 *l.* 1 *s.* 3 *d.* the money that 326 ¼ ℔ of tobacco will come to, at the rate of 3 *s.* 6 *d.* for 1 ½ ℔.

QUESTION II.

What is the worth of 19 *oz.* 3 *dwts.* 5 *grs.* of gold, at 2 *l.* 19 *s.* ⅌ *ounce.* ?

If 1 *oz.* — 2 *l.* 19 *s.* 4 *d.* — 19 *oz.* 3 *dwts.* 5 *grs.*

```
24 { 4 | 5
   { 6 | (125
20     | 3,2083
       | 0,1604166
```

Or thus in Decimals:

If 1 *oz.* —— 2,96 *l.* —— 19,1604166 *oz.*

```
                   19,1604166
                        2,966
                 ------------
              9) 114962500
                 ------------
                   1277361111
                 1724437500
                 3832083333
                 ------------
                 56,8425694
```

Becauſe the firſt term here is unity or 1, which neither multiplies nor divides, therefore the anſwer is produced by multiplying the ſecond and third terms together; and the product being valued, as ſhewn in Sect. II. gives 56 *l.* 16 *s.* 10 ¼ *d.* the value of the gold, as ſought after.

QUESTION III.

What is the worth of 827 $\frac{3}{4}$ *yards* of painting at 10 $\frac{1}{2}$ *d.* ℘ *yard.*

If 1 *yd.* —0,04375 *l.* — 827,750 0 *yd.*

57 340,0

33 1100 0
2 483 2
579 4
41 4

36,214 0 } Anſw. 36 *l.* 4 *s.* 3 $\frac{1}{2}$ *d.*

Now becauſe there would be 7 decimal places in the anſwer, whereof 4 are more than ſufficient, therefore to get 4 decimals in the product, the place of 0 units is put under the fourth decimal in the multiplicand, and the order of the reſt inverted as directed in Sect. IV.

QUESTION IV.

Lent my friend 34 *l.* for $\frac{5}{8}$ of a year, how much ought he to lend me $\frac{9}{12}$ of a year, to requite my kindneſs?

0,75 *y.* — 34 *l.* — 0,625 *y.*

34

2500
1875

75) 21,25 (28,333 Anſwer. 28 *l.* 6 *s.* 8 *d.*
625

250

25

QUES-

QUESTION V.

If $\frac{3}{4}$ of a yard of cloth, that is 2 $\frac{1}{4}$ yards broad, make a garment; how much of another ſort, that is but $\frac{4}{5}$ of a yard wide, will make the ſame garment?

,75

0,8 *y. b.*—0,75 *y. l.* — 2,25 *y. b.*

1125

1575

,8) 1,6875 (2,1093, *&c.*

Anſwer, 2 yards and 1 nail.

QUESTION VI.

If when the buſhel of wheat coſt 4*s.* 9*d.* the penny loaf weigh'd 10 $\frac{2}{3}$ ounces; what ſhould it weigh, when the buſhel of wheat is ſold for 8*s.* 10*d.*

£. — *oz.* — £.

0,441$\cancel{6}$ — 10,$\cancel{6}$ — 0,2375

10,$\cancel{6}$

9)14250

15833

23750

0,441$\cancel{6}$)2,5$\cancel{3}$

441) ,25

0,3975) 2,2800(5,72, *&c. Ounces*

2925

143

The foregoing method includes both the direct and inverſe rules of three: But the following rule, comprehends all caſes that come under the rules of three, whether direct or inverſe, whether ſingle or double, or any how compounded; and is of ſuch general uſe, as to extend to all arithmetical operations, where proportions are concerned: It was reduced to this form about forty years ago, by *William Jones*, Eſq; F. R. S. and has ever ſince been commonly known, and made uſe of by mathematicians.

The Rule of Proportion.

1. *Set down the terms expreſſing the condition of the queſtion, in one line.*
2. *Under each conditional term, ſet its correſponding one, in another line.*
3. *Multiply the producing terms of one line, and the produced term of the other line, continually; and take the reſult for a dividend.*
4. *Multiply the remaining terms continually, and let the product be a diviſor.*
5. *The quotient of this diviſion, will be the term required.*

By producing terms, here, is meant, whatever neceſſarily and jointly produce any effect; as, the cauſe and the time; length, breadth and depth; buyer and his money; ſeller and his goods; things carried and their diſtance; exchanger and the things exchanged, &c. all neceſſarily inſeparable in producing their ſeveral effects.

In a queſtion where a term is only underſtood, and not expreſſed, that term may ever be expreſſed by unity.

A term expreſſed by a number having different names, muſt be reduced ſo as to have the ſame name.

A quotient is repreſented by the dividend put above a line, and the diviſor put below it.

EXAMPLES.

I. If 150 £. ſerve 4 perſons for 16 weeks: What ſum will ſerve 13 perſons for 49 weeks?

£. *P.* *W.*
150 --- 4 --- 16
Q. --- 13 --- 49

$$\text{Now } Q. = \left(\frac{150 \times 13 \times 49}{4 \times 16} = \right) \overset{£.}{1492}\ \overset{s.}{19}\ \overset{d.}{4\tfrac{1}{2}}$$

For 150
13
450
150
1950
49
16 17550
4 7800
64) 95550(1492,96875 £.
315
595
190
620
440
560
480
320

II. If 40 acres of graſs be mowed by eight men in ſeven days: How many acres can be mowed by twenty-four men in twenty-eight days?

A.	M.	D.
40	8	7
Q.	24	28

Now $Q. = \left(\frac{40 \times 24 \times 28}{8 \times 7} =\right)$ 480 acres

III. If the carriage of 126 ℔ for 200 miles, coſt 6 ſhillings: How many ℔ will be carried 750 mile for 20 ſhillings?

℔.	M.	S.
126	200	6
Q.	750	20

Now $Q. = \left(\frac{126 \times 200 \times 20}{750 \times 6} =\right)$ 112 ℔.

IV. If 14 horſes in 16 days eat 56 buſhels of oats: How many buſhels will 20 horſes eat in 24 days?

H.	D.	B.
14	16	56
20	24	Q.

Now $Q. = \left(\frac{20 \times 24 \times 56}{14 \times 16} =\right)$ 120 buſhels.

V. How many yards of bays of 3 quarters wide, will ſuffice to line 1000 Soldiers coats, each containing 2½ yards of cloth, of 5 quarters wide?

yds long	qrs wi.	coat
2,5	5	1
Q.	3	1000

$$\text{Now } Q. = \left(\frac{2{,}5 \times 5 \times 1000}{3} = \right) 4166{,}\cancel{6} \text{ yds.}$$

VI. What is the intereſt of 542 £. 10 *s.* for 219 days, at the rate of 5 £. ꝑ *cent.* ꝑ *annum?*

£. prin.	*Days*	*£. int.*
100	365	5
542,5	219	Q.

$$\text{Now } Q. = \left(\frac{542{,}5 \times 219 \times 5}{100 \times 365} = \right) 16\,£.\ 5\,s.\ 6\,d.$$

VII. A footman can run 240 miles in 4 days, of 12 hours long: How many days of 16 hours long, will he be in running 720 miles?

Footm.	Da.	Ho.	miles
1	4	12	240
1	Q.	16	720

$$\text{Now } Q. = \left(\frac{4 \times 16 \times 720}{12 \times 240} = \right) 9 \text{ days.}$$

VIII. If 12 meaſures of wine, at 20 *d.* each, ſerve 8 men for 3 days: How many meaſures at 16 *d.* each, will ſerve 24 men for 2 days?

Mea.	Pence	M.	Da.
12	20	8	3
Q.	16	24	2

$$\text{Now } Q = \left(\frac{12 \times 20 \times 24 \times 2}{16 \times 8 \times 3} = \right) 30 \text{ meaſ.}$$

IX. If a garriſon of 3600 men have bread for 35 days, at 24 oz. each a day: How much a day may be allowed to 4800 men, each for 45 days, that the ſame quantity of bread may ſerve?

Men	oz.	da.	Bread.
3600	24	35	1
4800	Q.	45	1

$$\text{Now } Q = \left(\frac{3600 \times 24 \times 35}{4800 \times 45} = \right) 14 \text{ oz. ꝑ day.}$$

X. If when the tun of wine is worth 30 £; 20 £ worth will ſerve a ſhip's company of 336 men for 4 days, at a pint to each a day: How long will 500 £ worth ſerve a crew of 250 men, at $1\frac{1}{2}$ pint to each man a day; when the tun is worth 24 £.

Tun	£.	M.	da.	Pint	£.*w.*
1	30	336	4	1	20
1	24	250	Q.	1,5	500

$$\text{Now } Q = \left(\frac{30 \times 336 \times 4 \times 500}{24 \times 250 \times 1{,}5 \times 20} = \right) 112 \text{ days.}$$

XI. If when the quarter of wheat is ſold for 2 *£*. 12 *s*. 6 *d*. the three penny loaf weighs 22 *oz*. 16 *dwts*. What ought the 2 penny loaf to weigh, when the buſhel is ſold for 5 ſhillings?

Now 2 *£*. 12 *s*. 6 *d*. = 2,625 *£*.; and 22 *oz*. 16 *dwts*. = 22,8 *oz*.

Quar.	£.	pen. lo.	oz.
1	2,625	3	22,8
1	2,	2	Q.

Then $Q. = \left(\frac{2,625\times22,8\times2}{2\times3}=\right)$ 19,95 oz.

XII. If 48 men in 5 $\frac{1}{2}$ days, dig a trench of 23 $\frac{1}{4}$ yards long 2 $\frac{1}{3}$ deep, and 3 $\frac{2}{3}$ wide: What length of trench, of 3 $\frac{1}{2}$ yards deep, and 5 $\frac{3}{5}$ wide, can be dug by 24 men in 189 days?

Men	da.	yds lo.	deep	wide
248	5,5	23,25	$2,\acute{3}$	$3,\acute{6}$
24	189	Q.	3,5	5,6

Now $Q. = \left(\frac{24\times189\times23,25\times2,\acute{3}\times3,\acute{6}}{248\times5,5\times3,5\times5,6}=\right)$ 33,75 yds lo.

XIII. If 14 yards of cloth coſt 10 guineas: How many ells flemiſh may be had for 383 £. 17 *s.* 6 *d.* ?

Man	£.	Ell Fl. *
1	10,5	$14\times1,\cancel{3}$
1	383,875	Q.

$$\text{Then } Q.=\left(\frac{14\times1,\cancel{3}\times383,875}{10,5}=\right)\ \overset{\pounds.}{504}\ \ \overset{s.}{13}\ \ \overset{d.}{4}$$

XIV. If 13 ells of diaper of $\frac{3}{4}$ yard wide, coſt 5 guineas: What will 32 $\frac{1}{4}$ yards of $\frac{2}{3}$ ell engliſh wide, and of the ſame goodneſs, come to?

Man	£.	† yds. lo.	wide
1	5,25	$13\times1,25$	0,75
1	Q.	32,25	$0,\cancel{6}\times1,25$

$$Q.=\left(\frac{32,25\times0,\cancel{6}\times1,25\times5,25}{13\times1,25\times0,75}=\right)\ \overset{\pounds.}{8}\ \ \overset{s.}{12}$$

---

* *Note*, Yds multipl. by $\left(\frac{4}{\cancel{3}}=\right)$ $1,\cancel{3}$. gives ells flem.

† *Note*, ells engl. multip. by $\left(\frac{5}{4}=\right)$ 1,25 gives yds.

XV. One who had ſold a parcel of cloths at 2*s.* 10*d.* a yd on 3 mon. credit, found he had gain'd 25£. ℘ *cent.* ℘ *annum:* What did the cloth coſt a yard?

£. mon. £.
100 - - - 12 - - - 25
100 - - - 3 - - - Q.

$$\text{Now } Q = \left(\frac{100 \times 3 \times 25}{100 \times 12} = \right) 6{,}25\ \text{£.}$$

And £. 100 + £. 6,25 = £. 106,25. Alſo, 2 *s.* 10 *d.* = $0{,}141\cancel{6}$ £.

£. Mon. £.
Then 106,25 - - - 3 - - - 100
$0{,}141\cancel{6}$ - - 3 - - - Q.

$$\text{Therefore } Q = \left(\frac{0{,}141\cancel{6} \times 3 \times 100}{106{,}25 \times 3} = \right) \overset{\text{£.}}{0{,}1\cancel{3}}$$

Or, 2*s.* 8*d.* a yard is the prime coſt.

XVI. At 3*s.* 4*d.* a pound: What will 754 ½ ℔ be worth, allowing 4 ℔ upon every 100 pound?

Man ℔ ℔
1 - - - 104 - - - 100
1 - - - 754,5 - - Q.

$$\text{Now } Q = \left(\frac{754{,}5 \times 100}{104} = \right) 725{,}4807 \text{ ℔}$$

Man ℔ £.
1 - - - - 1 - - - - - - - $0{,}1\cancel{6}$
1 - - - 725,4807 - - - Q.

£. *s.* *d.*

$$\text{Then } Q = (725{,}4807 \times 0{,}1\cancel{6} =)\ 120\ \ 18\ \ 3\tfrac{1}{2}$$

XVII. One would exchange 729 pieces of 4*s.* 2*d.* each, for pounds ſterling, and muſt allow the broker 1¼ £. upon 100 £. How many £. will he receive?

Man Piece £.
1 - - - 1 - - - $0{,}208\cancel{3}$
1 - - 729 - - Q. $= 729 \times 0{,}208\cancel{3}$ £.

Man £. £.
1 - - - 101,25 - - - 100
1 - - - $729 \times 0{,}208\cancel{3}$ - - - Q.

$$Q = \left(\frac{729 \times 0{,}208\cancel{3} \times 100}{101{,}25} = \right) 150 \text{ £.}$$

XVIII. A draper bought 27 pieces of cloth, each of 24 ½ yards long, and 7 quarters wide, at 14 *s.* 8 *d.* a yard: How many pieces of ſuch cloth, each of 31 ¼ yards long, and 5 quarters wide, may be bought for 1375 £.?

yd lo. q. w. £.
1 - - - - - - 7 - - - 0,7$\cancel{3}$
27×24,5 - - 7 - - - Q. $=27\times24{,}5\times0{,}7\cancel{3}$ £.

Pieces yds long q. w. £.
27 - - - 24,5 - - - 7 - - - 27×24,5×0,7$\cancel{3}$
Q. - - - 31,25 - - 5 - - - - - - 1375.

Now $Q.=\left(\frac{7\times1375}{31{,}25\times5\times0{,}\cancel{7}\cancel{3}}=\right)$ 84 Pieces.

XIX. A grocer bought 4 ¼ hundred weight of pepper, for 15 £. 17 *s.* 4 *d.* which proving to be damaged, he is willing to loſe 12 ½ £. ℘ *cent.* How muſt he ſell it a pound?

Man £. £.
1 - - - 100 - - - 87,5
1 - - - 15,8$\cancel{6}$ - - Q.

$$Q.=\left(\frac{15{,}8\cancel{6}\times87{,}5}{100}=\right)\overset{£.}{15{,}8\cancel{6}}\times0{,}875$$

Man ℔ £.
1 - - 476 - - - 15,8$\cancel{6}$×0,875
1 - - 1 - - - - - - - - Q. $=\frac{15{,}8\cancel{6}\times0{,}875}{476}$ £.

He muſt ſell it at 7 *d.* a ℔.

XX. A merchant ſent to his factor at *Liſbon* 36 cloths, each of 22 ½ yards, at 9 *s.* 6 *d.* a yard; who is to return the third part in ſugar at 22 *s.* 6*d.* a hundred weight, and the reſt in wine, at 13 £. 10 *s.* a pipe: What quantity of ſugar and wine did the factor ſend?

Man yd £.
1 - - - - - - 1 - - 0,475
1 - - - 22,5×36 - Q.

$$Q. = (36 \times 22{,}5 + 0{,}475 =)\ 384{,}75\ £.$$

Then 384,75 £. is the value of the cloth.
And 128,25 £. is the ſum returned in ſugar.
Alſo 256,5 £. is the ſum returned in wine.

Man £. hun.wt.
1 - - 1,125 - - 1

$$1 - - 128{,}26 - - Q. = \left(\frac{128{,}25}{1{,}125} =\right) \text{114 hundr. wt. of ſugar.}$$

Man £. Pipe
1 - - 13,5 - - 1

$$1 - - 256{,}5 - - Q. = \left(\frac{256{,}5}{13{,}5} =\right) \text{19 pipes of wine.}$$

SECTION IX.

FOR the ſpeedy working of thoſe examples in the rule of three, where the firſt term is an unit; there are, in books of arithmetic, ſeveral compendious Rules, called the *Rules of Practice*, which are nothing more than the application of the doctrine of *aliquot parts:* But as there are a great variety of ſuch parts; ſo many, therefore, are the ways of applying them; which occaſions ſuch a diverſity of Rules for doing one and the ſame thing, that it would be an endleſs task, to give all the eaſy methods of operation adapted to particular Caſes; in this place it may ſuffice to mention ſome general directions, whereby, with the judgment of the practitioner, the moſt common Caſes may expeditiouſly be ſolved.

One number is ſaid to be an *aliquot part* of another; when the former will divide the latter, and leave no remainder.

Tables, ſhewing the aliquot parts of a pound ſterling, and of a ſhilling.

	Of a pound ſterling.		Of a ſhilling.		Of a pound.
10 *s.* 0 *d.* is	$\frac{1}{2}$	6 *d.* is	$\frac{1}{2}$	Or,	$\frac{1}{40}$
6 *s.* 8 *d.*	$\frac{1}{3}$	4 *d.*	$\frac{1}{3}$		$\frac{1}{60}$
5 *s.* 0 *d.*	$\frac{1}{4}$	3 *d.*	$\frac{1}{4}$		$\frac{1}{80}$
4 *s.* 0 *d.*	$\frac{1}{6}$	2 *d.*	$\frac{1}{6}$		$\frac{1}{120}$
3 *s.* 4 *d.*	$\frac{1}{5}$	$1\frac{1}{2}$ *d.*	$\frac{1}{8}$		
2 *s.* 6 *d.*	$\frac{1}{8}$	$\frac{3}{4}$ *d.*	$\frac{1}{16}$		
2 *s.* 0 *d.*	$\frac{1}{10}$				

The ſolution of all the Caſes in Practice, may be perform'd by the following,

General RULE.

Of the greater denominations of the things given to be valued.

1. Take the product thereof by the pounds in the price.
2. And alſo, the parts thereof, for the moſt convenient aliquots of the lower denominations of the price.
3. And the parts of the given price, for the moſt convenient aliquots of the lower denominations of the quantity.
4. The ſum of theſe, is the value of the quantity given.

In the following Examples; the aliquot parts, or diviſors, are ſet againſt the numbers they are to divide.

Ex. I. At 3 *s.* 6 *d.* a pair; what will 273 pair come to?

s.	*d.*				
2	0 is	$\frac{1}{10}$	273		
1	0	$\frac{1}{2}$	27	6	
0	6	$\frac{1}{2}$	13	13	
			6	16	6
		£.	47	15	6

Ex. II. At 17 *s.* 10 *d.* ½ a yard; what will 483 ¾ yards comes to?

```
            s.   d.
           10    0    is | ½  |483
                         |    |——————————————
            5    0       | ½  |241  10
            2    6       | ½  |120  15
            0    3       |¹⁄₁₀| 60   7   6
y.          0    1½      | ½  |  6   0   9
       ——————————————————|    |
½  | ½ |   17   10½           |  3   0   4½
   |   |————————————
¼  | ½ |    8   11¼  -  -  -  -  8  11¼
   |   |    4    5⁵⁄₂  -  -  -  -  4   5½
                              ——————————————
                              432   7   0¼
                              ——————————————
```

Ex. III. At 48 £. 14 *s.* 7 *d.* ¾ ℔ *C.wt.*; what will 596 *C.wt.* 3 *qrs.* 19 ℔ ½ come to?

```
           £.   s.   d.
            0   10    0 is | ½ |    596
                           |   |     48
                           |   |————————
                           |   |   4768
                           |   |  2384
                           |   |————————
           48    0    0    |   |  28608
            0    2    0    | ⅕ |    298 the ½ of 596
            0    2    0    |¹⁄₁|  - -59  12
            0    0    6    | ¼ |  - -59  12
            0    0    1½   | ¼ |  - -14  18
qrs. ℔      0    0    0¼   | ⅙ |  - - -3  14  6
 2    0 is| ½ |48   14    7¾ |   |  - - -0  12  5
 1    0   | ½ |  -  -  -  -  -     24   7  3¾,5
 0   14   | ½ |  -  -  -  -  -     12   3  7¾,75
 0    2   | ⅐ |  -  -  -  -  -      6   1  9¾,87
 0    2   |¹⁄₁|  -  -  -  -  -      0  17  4¾,41
 0    1   | ½ |  -  -  -  -  -      0  17  4¾,41
 0    0½  | ½ |  -  -  -  -  -      0   8  8¼,7
                                    0   4  4 ,85
                                ——————————————————
                             £. 29089   9  7 ,49
                                ——————————————————
```

But moſt Examples of this kind, are more readily ſolved, by mixing the doctrine of decimal fractions, with that of aliquot parts; as ſhewn by the following,

General RULE.

1. To the greater denomination of the things given to be valued, annex the decimal of the inferior parts (if any.)
2. Take the product of this mix'd number, by the pounds in the price.
3. And alſo, the product of $\frac{1}{10}$ thereof, by half the greateſt even ſhillings in the price.
4. Out of the ſaid $\frac{1}{10}$ of the things given to be valued, take the moſt convenient aliquots of 2 ſhillings, for the other parts (if any) of the price.
5. The ſum of the lines produced by the 2d, 3d, and 4th articles, will be the anſwer decimally expreſſed.

Ex. I. At 3 *s.* 6 *d.* a pair; what will 273 pair come to?

27,3 is $\frac{1}{10}$ of 373
s. d. 1 is $\frac{1}{2}$ the greateſt even ſhillings
1 0 | $\frac{1}{2}$ | 27,3 the value at 2 ſhillings
0 6 | $\frac{1}{2}$ | 13,65 the value at 1 ſhilllings
6,826 value at 6 pence
£. 47,775 = 47 £. 15 *s.* 6 *d.*

Ex. II. At 17 *s.* 10 *d.* $\frac{1}{2}$ a yard; what will 483 $\frac{3}{4}$ yards come to?

Now 483 $\frac{3}{4}$ is 483,75

s. d.			
1 0	$\frac{1}{2}$	48,375	is $\frac{1}{10}$ of 483,75
		8	is $\frac{1}{2}$ even ſhillings
		387,000	is the value at 16 ſhillings
0 6	$\frac{1}{2}$	24,1875	is the value at 1 ſhilling
0 3	$\frac{1}{2}$	12,09375	is the value at 6 pence
0 1$\frac{1}{2}$	$\frac{1}{2}$	6,04687	is the value at 3 pence
		3,02343	is the value at 1 $\frac{1}{2}$ *d.*
		£. 432,35155	

Ex. III. At 48 £. 14 *s.* 7 $\frac{3}{4}$ ⅌ *C.wt.*: What will 596 *C.wt.* 3 *qrs.* 19 $\frac{1}{2}$ ℔ come to?

	2	1	
	28 { 4	19,5	
	7	4,875	
s. d.	4	3,69643	
0 6	$\frac{1}{4}$	596,92411	its $\frac{1}{10}$ is 59,692411
		48	
		477539288	
		238769644	
		28652,35728	
		417,846877	is 59,692411 by 7
0 1$\frac{1}{2}$	$\frac{1}{4}$	14,923103	is $\frac{1}{4}$ of 59,69, &c.
0 0$\frac{1}{4}$	$\frac{1}{6}$	3,730776	
		0,621796	
		29089,479832	

This Rule ſaves the trouble of taking the parts of the price, ſor the parts of the given things.

SEC-

SECTION X.

Of Powers and their Roots.

WHEN a given number is multiplied by itſelf, and this product by the given number, and this product by the given number, &c. to any aſſigned number of products; this proceſs is called the involution of the given number, or the raiſing it to its powers.

Thus, the given number, is called the root or firſt power.

The 1ſt pow. multi. by itſelf, gives the 2d pow.
The 2d pow. multi. by the 1ſt, gives the 3d pow.
The 3 pow. multi. by the 1ſt, gives the 4th pow.
&c.

The 2d power is called the ſquare.
The 3d power is called the cube.
The 4th power is called the biquadrat.
&c.

The following table exhibits the 1ſt 2d and 3d powers of the nine digits.

Roots.	1.	2.	3.	4.	5.	6.	7.	8.	9.
Squares.	1.	4.	9.	16.	25.	36.	49.	64.	81.
Cubes.	1.	8.	27.	64.	125.	216.	343.	512.	729.

From what has been ſaid, it will be eaſy to find any aſſigned power of a given number: but when a large number is conſidered as a given power, and its root be required, this is not to be done ſo readily as the raiſing the power from the given root; in the latter Caſe, the factors are always known; but in the former, there is given, only the dividend, the diviſor to which, is to be found; and this is not conſtant, as in Diviſion, but changes every time a new figure or place is obtained in the quotient.

The method of finding the roots of given powers, is called, the Extraction of roots.

The common methods of performing the operations in the ſquare and cube roots only, will here be ſhewn; thoſe of higher powers not being much wanted in common menſuration: but the more inquiſitive ſort of readers may be amply ſatisfied in the methods of extracting the roots of higher powers, in almoſt every book of elementary Algebra.

To extract the ſquare root of any given number.

RULE.

1. Put a point over the place of units; and alſo, over every ſecond place (counting from units,) to the left-hand, for integers, and to the right-hand, for decimals; and the integral part of the root, will have as many places, as there are points over the integers in the given number.

When a number is thus pointed, the place under a point, and its left-hand place together, is called a period.

2. Seek the greateſt ſquare in the left-hand period, write the root in the quotient; the ſquare thereof, write under the period; ſubtract, and to the remainder bring down the next period; (as in Diviſion,) call this the reſolvend.

3. To the left of the reſolvend, write the double of the quotient for a diviſor; ſeek how oſt this may be had in the reſolvend, except its right-hand place; write the reſult in the quotient; and alſo on the right of the diviſor.
4. Multiply this increas'd diviſor, by the laſt quotient figure; ſubtract the product from the reſolvend; to the remainder bring down the next period for a new reſolvend; double all the quotient for a diviſor; divide as before; and thus proceed until all the periods are uſed.

If at laſt there happen to be a great remainder, and it is required to have the root more accurate, by increaſing it with a decimal fraction. To the remainder annex two cyphers, and proſecute the work as before, always adding two cyphers to the remainder, &c. till the root is as exact as deſired.

E X A M P L E I.

What is the ſquare root of 132496?

```
        132496 (364
         9
       ------
  66)   424
        396
       ------
 724)   2896
        2896
       ------
```

The firſt period towards the left-hand is 13, the greateſt ſquare there in is 9, whoſe root 3, write in the quote, and the ſquare 9 write under the 13, and ſubtracting, there remains 4; to which bring down the next period 24, makes the reſolvend 424; to the left-hand thereof draw a curv'd line, and at ſome diſtance therefrom put the double of 3, *viz.*

viz. 6, and inquire how oft this 6 may be had in 42, and find 6 times, write 6 in the quote, and alſo, on the right-hand of the diviſor 6; and the increaſed diviſor 66, multiply by the 6 in the quote, and it gives 396, this ſubtracted from 424 leaves 28; to which, the next period 96 is brought down, and 2896 is the new reſolvend; now 36 the quotient, doubled, makes 72 the diviſor, this in 289 goes 4 times, write 4 in the Root, and diviſor; and the new diviſor 724, multiplied by 4, gives 2896, to be ſubtracted from the laſt reſolvend, and nothing remains: therefore 364 is the true root; for 364 multipled by 364, gives 132496 the number given.

EXAMPLE II.

What is the ſquare root of 763958207163?

763958207163) (874047,027
64
167) 1239
1169
1744) 7058
6976
174804) 822071
699216
1748087) 12285563
12236609
174809402) 489540000
349618804
1748094047) 139921119600
122366658329
1755461281, *&c.*

In this Example, after all the periods in the given number are brought down and uſed; to the remainder are brought down periods of cyphers; and with theſe, the Work is proſecuted in the ſame manner as if they were given periods of ſignificant digits: And thus, may the root be continued to almoſt any deſired exactneſs; for there will ever be a remainder, ſince no digit ſquared, can have a cypher in its right-hand place.

More EXAMPLES.

What is the ſquare root of

	Anſwer.
36372961	6031
24681024	4968
1,0609	1,03
911236798,794365	30186,699

Theſe Examples are operated in the ſame manner as the preceding ones.

There are many uſes to which the ſquare root may be applied, one is,

Two numbers being given, between them to find a mean proportional.

RULE.

Multiply the two numbers together, and out of the product extract the ſquare root, which root is the mean proportional required.

EXAMPLE I.

What is the mean proportional between 3 and 12?

Now 3 multiplied by 12, is 36; whoſe ſquare root is 6, the mean required.

For 3 is to 6, ſo is 6 to 12, by the rules of proportion.

EXAMPLE II.

Find a mean proportional between 4276 and 842?

```
   4276             ·  ·  · ·
    842             3600392 (1897,4, &c.
  -----             1
   8552        28) 260
  17104            224
 34208       369) 3603
 -------           3321
 3600392     3787) 28292
                   26509
           3794,4) 178300
                   151776
                    26524, &c.
```

So 1897,4, &c. is the mean proportional required.

To extract the cube root of a given number.

RULE.

1. Put a point over the place of units; and alſo, over every third place, counting to the left for integers, and to the right for decimals; or in other words, point the given number into periods of three places each, beginning at units: and there will be as many integral places in the root, as there are points over the integers in the given number.

2. Seek the greateſt cube in the left-hand period, write the root in the quotient, and the cube under the period; ſubtract, and to the remainder bring down the next period: Call this the Reſolvend, under which draw a line.

3. Under the Reſolvend, write the triple ſquare of the root, ſo that units in the latter ſtand under the place of hundreds in the former; under the triple ſquare of the root, write the triple root, removed one place to the right; and the ſum of theſe two lines call a Diviſor; under which draw a line.

4. Seek how oft this Diviſor may be had in the Reſolvend, [its right-hand place excepted,] and write the reſult in the quotient.

5. Under the Diviſor, write the product of the triple ſquare of the root by the laſt quotient figure, ſetting units place of this line, under that of tens in the Diviſor; under this line write, the product of the triple root, by the ſquare of the laſt quotient figure, let this line be removed one place beyond the right of the former; and under this line, removed one place forward to the right, write the cube of the laſt quotient figure; the ſum of theſe three lines call the Subtrahend, under which draw a line.

6. Subtract the Subtrahend from the Reſolvend; to the remainder bring down the next period for a new Reſolvend; the Diviſor to this, muſt be the triple ſquare of all the quotient added to the triple thereof, *&c.* as in the 3d article, *&c.*

EX-

EXAMPLE. I.

What is the Cube Root of 48228544?

48228544 (364

	27		
	21228	Reſolvend.	
add	27	Triple ſquare of 3.	the root.
	09	Triple of 3. - -	
	279	Diviſor.	
add	162	Triple ſquare of 3 multiplied by 6.	
	324	Triple of 3 multipl. by ſquare of 6.	
	216	Cube of 6.	
	19656	Subtrahend.	
	1572544	Reſolvend.	
add	3888	Triple ſquare of 36.	the root.
	108	Triple of 36. - -	
	38988	Diviſor.	
add	15552	Triple ſquare of 36 mult. by 4.	
	1728	Triple of 36 multi. by ſquare of 4.	
	64	Cube of 4.	
	1572544	Subtrahend.	

If the work of this Example be well conſidered, and compared with the foregoing Rule, it will be eaſy to conceive how any other Example of the like nature may be wrought; and here obſerve that when the cube root is extracted to more than two places, there is a neceſſity of doing ſome work upon a ſpare piece of paper, in order to come at the root's triple ſquare, and the product of the triple root by the ſquare of the quotient figure, *&c.*

In this example, the given number is a cube number, and therefore at the end of the operation there remained nothing; for 364 multiplied by 364, the product multiplied by 364 gives 48228544, the given number.

But if the number given be not a cube number; then, to the laſt remainder always bring down three cyphers, and work anew for a decimal fraction if needful.

More EXAMPLES.

What is the cube root of

	Anſwers.
389017	73
1092727	103
27054036008	3002
219365327791	6031
122615327232	4968

Theſe examples are all operated in the ſame manner as the foregoing one.

There are many uſes of the cube root, one is to find two mean proportionals between two given numbers.

RULE.

Divide the greater extream by the leſſer, and the cube root of the quotient multiplied by the leſſer extream, gives the leſſer mean. Multiply the ſaid cube root by the leſſer mean, and the product is the greater mean proportional.

Note, This is only underſtood of thoſe numbers that are in continued geometric proportion.

EXAMPLE. I.

What are the two mean proportionals between 4 and 108?

108 divided by 4 gives 27, whoſe cube root is 3; and the leſſer extream 4, multiplied thereby, gives 12 for the leſſer mean; and 12 multiplied by the ſaid root 3, gives 36 for the greater mean.

For as 4 is to 12, ſo is 36 to 108.

EXAMPLE II.

Find the two geometric means between 8 and 1728?

Now 8) 1728 (216, whoſe cube root is 6. And 6 times 8 is 48, the leſſer mean, and 6 times 46 is 288 the greater mean.

For as 8 is to 48 ſo is 288 to 1728.

If the rule already given for the cube root be thought too tedious, the following one will be found more ready for uſe.

1. Point the given number, ſeek the greateſt cube in the left-hand period, write the root in the quotient, ſubtract the cube from the period, as directed in the other rule; and to the remainder bring down all the remaining periods in the given number: Call this the Reſolvend.

2. To the root (or quotient) annex as many cyphers, as there are remaining periods; multiply this by 3; by this product, divide the Reſolvend; and point the quotient into periods of 2 places, (beginning at units,) obſerving that there be no more points than there were periods brought down to the Reſolvend.

3. Make the root (found in the firſt period of the given number,) a Diviſor, ſeek how often it may be had in the left hand period of the quotient [excepting the place under the point] and the figure

reſulting write in the quotient, [to the right-hand of the root firſt found] and on the right of the diviſor; multiply this increaſed diviſor by the laſt quotient figure; to the remainder bring down the next period; divide this, by the laſt diviſor, &c.

An example or two will render the whole very plain.

What is the cube root of 12812904?

```
   .  .  .
   12812904   (200
    8            3
   ----------------
6,00) 4812904   600
      -------
       .  .
  23) 8021,5, &c.  (34
      69            200
      ------       ----
 234) 1121          234
       936
      ----
       185
```

First begin at 4 the place of units, over which put a point, and omitting two figures, put another point over the 2; and omitting two figures more, put another point over the left-hand figure 2: Now here are three points, and therefore there muſt be three places of integers in the root. Then, beginning with the firſt period 12, find the greateſt cube therein, which is 8, whoſe root is 2; write 8 under 12, and 2 as a quotient. Subtract 8 from 12, and to the remainder 4 bring down the remaining figures (which occupy the places of two points or periods.) To the quotient, (2) annex two cyphers, for the two points remaining over the given number, [for the quotient 2 is in reality 200,] and this 200, multiply by 3, and the product 600 make a diviſor; by it divide 4812904, (which is the difference between 8 the cube of 2, and the given number,) and the quotient is 8021,5, *&c.* Then begin at 1, the place of units, and point as directed in the ſquare, till there are as many points as annexed cyphers to the firſt root, which in this example are two. To the right and left of this number, *viz.* 8021, *&c.* draw curved lines, as in diviſion; make the firſt root 2 a diviſor, inquire how oft it may be found in the firſt period 80 [excepting the place under the point; that is, ſay how often 2 in 8,] and it gives 3, write this in the quotient; and alſo on the right-hand of the diviſor 2, which now becomes 23. This 23 multiply by the quotient 3, and the product 69, ſubtract from 80, to the remainder 11, bring down the next period 21, makes 1121. Now 23 the diviſor, in 112 goes 4, write 4 in the quotient, and on the right of 23, which now becomes 234; this 234 multiriplied dy the quotient 4, gives 936, and ſubtracting, there remains 187.

Now this quotient 34 added to the firſt root 200, makes 234, and if this 234 be cubed, it will be 12812904, which was the number firſt given, and therefore 234 is the true root required.

EXAMPLE II.

Extract the cube root out of 9239864750621, ſo that the root may conſiſt of eight Figures or Places.

```
              9239864750621 7.   (40000
              64                      3
              ---------------   ------
   12,0000)  2839864750,6217    120000
              ---------------
        45)  236655395          (52
              225                400, &c.
              ----
       425)  1165                452, &c.
              904
              ----
```

Then 452 cubed is 92345408. And the whole operation renewed, calling 452 the firſt root, will ſtand as follows;

```
               9239864750621 7      (45200
               92345408                  3
               ____________________  ______
                                     135600
 1356,00)  532395062,17  (392621,727293
   45208)  392621,727293 (08,684, &c.
           361664
           ______
  452086)   3095772          45200
            2712516            08,684
            _______          _________
 4520868)   38325672         45208,684
            36166944         the root.
            ________
45208684)   215872893
            180834736
            _________
             35038157, &c.
```

In this example, when three places were found in the root, theſe were conſidered as the root of the three firſt periods; therefore 452 cubed gives 92345408, which uſed as the firſt cube of 4 was, *viz.* ſubtracted from the given number 92398647506217, leaves 53239506217, this divided by the root 452 with two cyphers annexed, (for the remaining two points over the number given,) *viz.* 45200, gives 392621,727293, which pointed as in the ſquare, and the root 452 uſed as a diviſor, &c. as before directed, gives 08,684, this added to the foregoing root, 45200, gives 45208,684 for the root required.

This method very ſeldom fails to give the root of any number true to three places at leaſt, at the firſt operation; and if the ſecond place in the root is a cypher, or an unit, four or five places may be obtain'd at the firſt operation.

But if a ſecond operation is made, as in the foregoing example, eight or nine places in the root will be found: And if more accuracy be required, make a third operation, and this will give the root to 26 or 27 places; each operation tripling the figures found in the laſt root

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SECTION XI.

Of CHARACTERS *and their* EXPLANATION.

IN the following ſheets of this treatiſe, there are ſeveral characters and expreſſions uſed in order to ſhorten the work which are here explained.

I. Wherever is found this ſign +, *(more)* it ſignifies that the number following the ſign is to be added to the number going before it; thus 4 + 8 is read 4 more 8, and ſignifies, that 8 is to be added to 4.

II. This ſign — *(leſs)* ſignifies that the number following it, is to be ſubtracted from the number going before it; thus 6 — 2, is read 6 leſs 2, and ſignifies, that 2 is to be taken from 6.

III. This ſign × *(into)* ſignifies multiplication, and implies that the numbers this ſign is between, are to be multiplied together; thus 4 × 9 imports, that 4 is to be multiplied by 9; and 2 × 3 × 6 × 5, ſignifies that 2 is to be multipled by 3, and that product by 6, and this product by 5, and the like of any other.

IV. This ſign $\div$ *(by)* ſignifies diviſion, and ſhews that the number going before the ſign, is to be divided by the number following it; thus $12 \div 4$ implies, that 12 is to be divided by 4; but diviſion is moſt commonly expreſſed by ſetting down the dividend or number to be divided, and placing the diviſor or dividing number under it, with a line drawn between them, like a vulgar fraction; thus $\frac{12}{4}$ implies that 12 is to be divided by 4; and if 48,327 was to be divided by 2,15, expreſs it thus, $\frac{48,327,}{2,327,}$ *&c.*

V. This ſign $=$ *(equal)* ſignifies that the numbers or expreſſions on each ſide thereof, are equal one to the other; thus $4 + 8 = 12$, ſignifies that 8 added to 4 is equal to 12; and $6 - 2 = 4$, implies that 2 taken from 6 leaves 4, or 6 leſſened by 2 is equal to 4; and $4 \times 9 = 36$, implies that 4 multiplied by 9, gives a product equal to 36; and $\frac{12}{4} = 3$, ſignifies that 12 divided by 4, gives a quotient equal to 3, and the like of other expreſſions.

VI. The terms of proportions are expreſſed by certain points between the terms; thus 4 : 6 : : 10 : 15, and is read, as 4 is to 6, ſo is 10 to 15, ſo that the two points: between the two firſt terms is read, *is to*; the four points : : between the ſecond and third terms is read, *ſo is*, and the two points : between the third and fourth terms is read, *to*.

Take an example where all the forementioned characters are uſed: Suppoſe I buy 120 eggs at two a penny, and 120 more at three a penny, and ſell them again at five for two-pence; whether do I loſe or gain, and how much?

Now $\frac{120}{2} = 60$ *d.* the price the firſt 120 coſt; and $\frac{120}{3} = 40$ *d.* the price that the ſecond 120 coſt; and $60 + 40 = 100$, the price 240 coſt. Then as 5 *egg.* : 2 *d.* : : 240 *egg.* : 96 *d.*; for $240 \times 2 = 480$, and $\frac{480}{5} = 96$*d.* the price the eggs were ſold at; and 100 *d.* — 96 *d.* = 4 *d.* the money loſt.

This ſign $\surd$, ſhews that the ſquare root is to be found, of thoſe quantities affected with the ſign.

Thus $\sqrt{40 \times \frac{18}{2}}$, ſhews that 40 is to be multiplied by $\frac{18}{2}$, and the ſquare root of the product is to be taken.

The character $\sqrt[3]{\ }$, denotes the cube root of the quantities following; thus $\sqrt[3]{4 + \frac{3}{2}}$ ſhews that the half of 3 is to be added to 4, and the cube root of the ſum to be taken.

When ſeveral terms, or numbers, are connected together by lines drawn either above or below them; it implies that the reſult of thoſe terms or numbers, ordered as their ſigns denote, is to be taken as one term or number.

Thus $4 \times \overline{2 + 5}$, ſhews that the ſum of 2 and 5, or 7, is to be multiplied by 4, and makes 28: But was it wrote thus $4 \times 2 + 5$; it would denote, that 4 was to be multiplied by 2, and the product 8, added to 5, making 13; which is very different from the former reſult.

Again, $\overline{6 - \frac{\overline{3+5} \times 2}{4}} \times 7$ ſhews that the ſum of 3 and 5, or 8, is to be multiplied by 2; and the product 16, to be divided by 4; and the quotient 4, to be ſubtracted from 6; and the remainder 2, to be multiplied by 7; ſo that the whole expreſſion is equal to 14. But was it wrote thus $6 - \frac{3+5 \times 2}{4} \times 7$, the reſult would be very different; for it would denote, that the product of 5 by 2, or 10, was to be added to 3; and the ſum 13, to be divided by 4, and the quotient $3\frac{1}{4}$ to be multiplied by 7; and the product $22\frac{3}{4}$, to be ſubtracted from 6. Theſe things therefore ſhould be carefully attended to, for the drawing of a line, or leaving it out, makes a wide difference in the meaning of an expreſſion.

Several quantities, ſtanding under a line, with a figure at the right-hand end, ſhews that the expreſſion under that line, is to be rais'd to the power denoted by the index at the end of the line.

Thus $\overline{3 \times \frac{5}{4} + 6}^{2}$ denotes the ſecond power of this expreſſion, *&c.*

SECTION XII.

Of DUODECIMALS.

That ſcale of numeral notation, in which every ſuperior place, is twelve times its next inferior, is called duodecimals.

THIS way of conceiving the unit to be divided, is chiefly in uſe among workmen; and they uſe it only in caſting up the contents of their ſuperficial and ſolid works.

Artifices generally take the linear dimenſions of their work in feet, inches and parts.

And. 1 linear foot = 12 linear inches.
1 linear inch = 8, or 12 linear parts.
Alſo. 1 ſquare foot = 144 ſquare inches.
1 ſquare inch = 64, or 144 ſquare parts.
Again. 1 cubic foot = 1728 cubic inches.
1 cubic inch = 512 or 1728 cubic parts.

The difference in the parts, ariſes from conſidering the inch as divided into 8 parts or 12 parts: For ſome workmen take their dimenſions in feet, inches, and half quarters, or eighth parts: Others take them in feet, inches and quarters, and reckon every quarter as 3 parts; ſo that theſe actually follow the duodecimal ſcale; altho' they ſeldom take notice of any other parts of an inch, than 3, 6, or 9, and give and take (as they call it) for the intermediate ones.

The operation of multiplying feet and inches, by feet and inches; or, feet, inches and parts, by feet, inches and parts, is commonly called *croſs multiplication.*

As in decimals, the places deſcend by tens from the unit place to the right-hand, and are called primes, ſeconds, thirds, fourths, *&c.* ſo in duodecimals, the decreaſe is by twelves from the feet place to the right-hand, and are generally called feet, inches, parts, ſeconds, thirds, *&c.*

But as the terms, inches, parts, *&c.* are notions generally annexed to linear meaſure; and as the multiplication of theſe meaſures by one another produce denominations different from thoſe in the multiplying factors; therefore, it will be more convenient to call the terms, inferior to feet, primes, ſeconds, thirds, fourths, *&c.* and theſe will equally ſuit linear, ſuperficial and ſolid meaſures, juſt as well as the ſame names do in decimals, after any multiplication whatever.

And for diſtinction, let feet, be mark'd with (f;) primes, with (′;) ſeconds, with (″;) thirds, with (‴;) fourths, with (iv;) *&c.*

Theſe marks may very properly be called the indices of the terms to which they belong; that is, they ſhew how many terms diſtant they are from the place of feet.

RULES

RULES *for multiplying duodecimally.*

1. Under the multiplicand, write the correſponding denominations of the multiplier.

2. Multiply each term in the multiplicand, beginning at the loweſt, by the feet in the multiplier; write each reſult under its repective term; obſerving to carry an unit for every 12, from each lower denomination to its next ſuperior.

3. In the ſame manner, multiply all the multiplicand by the primes in the multiplier; and write the reſult of each term, one place removed to the right-hand of thoſe in the multiplicand.

4. Work in the ſame manner with the ſeconds in the multiplier, ſetting the reſult of each term, removed two places to the right-hand of thoſe in the multiplicand.

The ſum of theſe, gives the product required.

EXAMPLE I.

	f.	′	″	‴	IV	
Multiply	10	4	5			*the multiplicand* = *A*
By	7	8	6			*the multiplier* = *B*
Add	72	6	11	-	-	= A × 7f.
	6	10	11	4	-	= A × 8′
		5	2	2	6	= A × 6″
	79	11	0	6	6	= A × B = prod.

It amounts to the ſame, and is equally convenient, to begin firſt with the loweſt name in the multiplier; obſerving to place the reſults right.

$$\begin{array}{rrrrrrl}
 & \text{f.} & {}' & {}'' & & & \\
\text{Thus ex. 2. } \textit{mul.} & 7. & 8. & 6 & & & = A \\
\textit{By} & 10. & 4. & 5 & & & = B \\ \cline{2-6}
 & & 3. & 2. & 6. & 6 & = A \times 5'' \\
\text{Add}\left\{\vphantom{\begin{array}{c}1\\1\\1\end{array}}\right. & 2. & 6. & 10. & 0. & & = A \times 4' \\
 & 77. & 1. & 0 & & & = A \times 10\text{f.} \\ \cline{2-6}
 & \text{f.} & {}' & {}'' & {}''' & {}^{\text{iv}} & \\
 & 79. & 11. & 0. & 6. & 6 & = A \times B = \text{prod.} \\ \cline{2-6}
\end{array}$$

It will often happen, that the feet in the given dimenſions are ſo many, that to multiply them by the leſſer denominations, and to take a twelfth of their product, as above directed, will require ſome work to be done on a ſpare paper; to avoid which, obſerve the following,

RULE.

1. Divide the feet by 12, mentally reſerving the quotient and remainder.

2. Multiply the reſerved remainder by the leſſer denominations, writing the over plus of 12*s.* by each term, in their reſpective places.

3. Multiply the reſerved quotient, by thoſe leſſer denominations, adding thereto, the 12*s.* carried, and write the reſults in their proper places.

EXAMPLE III.

	f.	′	″	‴	iv	
Mul.	368	7	5			$=A$
By	137	8	4			$=B$
Add	10	2	10	5	8	$=A\times 4''$
	245	8	11	4	-	$=A\times 8'$
	4	9	1	-	-	$=137\text{f.}\times 5''$
	79	11	-	-	-	$=137\text{f.}\times 7'$
	2576	-	-	-	-	$\left.\right\} = 368\text{f.}\times 137\text{f.}$
	4783	-	-	-	-	
	50756	7	10	9	8	$=A\times B=$ produ.

	f.	quo. rem.
Now	$\frac{368}{12}=$	30 . 8
And	$\frac{137}{12}=$	11 . 5

EXAMPLE IV.

	f.	′	″	‴	iv	v	vi	
Mul.	24	10	8	7	5			$=A$
By	9	4	6					$=B$
Add	224	0	5	6	9	-	-	$=A\times 9\text{ f.}$
	8	3	6	10	5	8	-	$=A\times 4'$
	1	0	5	4	3	8	6	$=A\times 6''$
	233	4	5	9	6	4	6	$=A\times B=$ produ.

Ex.

Example 5. *Suppoſe the dimenſions of a block of marble, were*

```
             f.   ′   ″
Length.      9 .  6 .  5 ⎫
Breadth.     4 .  7 .  8 ⎬ required the ſolidity.
Depth.       2 . 10 .  3 ⎭
             f.   ′   ″
Multiply     9 .  6 .      = length  = A
By           4 .  7 .  8   = *breadth = B
                  6 .  4 .  3 .  4 = A × 8″
             5 .  6 .  8 . 11 .  - = A × 7′
            38 .  1 .  8 .  - .  - = A × 4f.
             f.   ′    ″    ‴   iv
And         44 .  2 .  9 .  2 .  4 = A × B
By           2 . 10 .  3           = depth = C
            88 .  5 .  6 .  4 .  8 .  - . - = A × B × 2f.
            36 . 10 .  3 .  7 . 11 .  4 . - = A × B × 10′
                 11 .  0 .  8 .  3 .  7 . 0 = A × B × 3″
             f.   ′    ″    ‴   iv    v   vi
Gives      126 .  2 . 10 .  8 . 10 . 11 . 0 A×B×C= ſolidity,
```

---

* That the ſeveral denominations ariſing from any multiplication (whether the factors have feet in them, or not) may be rightly eſtimated; the following rules will be found neceſſary.

RULES

RULE.

Feet by feet, give feet.
Feet by primes, give primes.
Feet by ſeconds, give ſeconds,
&c.

Primes by primes, give ſeconds.
Primes by ſeconds, give thirds.
Primes by thirds, give fourths,
&c.

Seconds by ſeconds, give fourths.
Seconds by thirds, give fifths.
Seconds by fourths, give ſixths,
&c.

Thirds by thirds, give fixths.
Thirds by fourths, give ſevenths.
Thirds by fifths, give eighths,
&c.

In general thus:

When feet are concerned, the product is of the ſame denomination with the term multiplying the feet.

When feet are not concerned, the name of the product will be expreſſed by the ſum of the indices of the two factors.

Theſe rules will be ſufficiently illuſtrated by the following example wrought at length.

	f.	$'$	$''$	$'''$	IV		
Multiply	24.	10.	8.	7.	5		
By	9.	4.	6				
Th. $24^{f.}\times 9^{f.}=$	216	--	--	--	--	--	--
$10'\times 9^{f.}=$	--	90	--	--	--	--	--
$8''\times 9^{f.}=$	--	--	72	--	--	--	--
$7'''\times 9^{f.}=$	--	--	--	63	--	--	--
$5^{\text{IV}}\times 9^{f.}=$	--	--	--	--	45	--	--
$24^{f}\times 4'=$	--	96	--	--	--	--	--
$10'\times 4'=$	--	--	40	--	--	--	--
$8''\times 4'=$	--	--	--	32	--	--	--
$7'''\times 4'=$	--	--	--	--	28	--	--
$5^{\text{IV}}\times 4'=$	--	--	--	--	--	20	--
$24^{f.}\times 6''=$	--	--	144	--	--	--	--
$10'\times 6''=$	--	--	--	60	--	--	--
$8''\times 6''=$	--	--	--	--	48	--	--
$7'''\times 6''=$	--	--	--	--	--	42	--
$5^{\text{IV}}\times 6''=$	--	--	--	--	--	--	30
ſum	216.	186.	256.	155.	121.	62.	30

Now $186'=$	15.	6					
$256''=$	1.	9.	4				
$155'''=$	-.	1.	0.	11			
121 IV=	--	--	--	10.	1		
62 V=	--	--	--	--	5.	2	
30 VI=	--	--	--	--	--	2.	6
	f.	$'$	$''$	$'''$	IV	V	VI
	233.	4.	5.	9.	6.	4.	6

f.

	f.	′	″	‴	IV	V	VI
=	216	-	-	-	-	-	-
=	7	6	-	-	-	-	-
=	-	6	0	-	-	-	-
=	-	-	5	3	-	-	-
=	-	-	-	3	9	-	-
=	8	-	-	-	-	-	-
=	-	3	4	-	-	-	-
=	-	-	2	8	-	-	-
=	-	-	-	2	4	-	-
=	-	-	-	-	1	8	-
=	1	-	-	-	-	-	-
=	-	-	5	-	-	-	-
=	-	-	-	4	-	-	-
=	-	-	-	-	3	6	-
=	-	-	-	-	-	2	6
	233	4	5	9	6	4	6

Many examples, to exerciſe duodecimals, will be given in what follows.

The reader will readily ſee, that the numbers in each line in this page, are reſpectively equal to thoſe in the oppoſite line in page 90.

A
TREATISE
OF
MENSURATION.

MENSURATION is the finding the contents of ſuperficies and ſolids; by having their dimenſions given in any ſort of meaſure; and conſiſts of three parts; *viz. lineal*, *ſuperficial*, and *ſolid*.

I. *Lineal meaſure*, is the meaſuring of lengths, and is uſed in taking the dimenſions of ſuperficial and ſolid figures.

II. *Superficial meaſure*, is the finding the number of ſquare inches, feet, yards, *&c.* in any plain or convex figure; as a floor, a wall, a globe, *&c.* by having the dimenſions in lineal meaſure.

III. *Solid meaſure*, is the finding the number of cubic, or ſolid inches, feet, yards, *&c.* in thoſe figures that have length, breadth, and depth; as a block of ſtone, marble, *&c.* by having the dimenſions in lineal meaſure.

Of linear meaſure it may ſuffice to obſerve, that

12 inches - are -	1 foot.
3 feet - - - - - -	1 yard.
6 feet - - - - -	1 fathom.
5½ yards, or 16½ feet -	1 pole, perch or rod.
40 poles - - - -	1 furlong.
8 furlongs - - - -	1 mile.

PART

PART I.

Of Superficial MEASURE.

Wherein will be ſhewn,

First, *Some uſeful definitions, and problems.*

Secondly, *The commom methods of calculating uſed by artificers.* And

Thirdly, *Methods of computing the areas of various ſorts of plane figures.*

SECTION I.

DEFINITIONS.

I. A Line is length without breadth; and is either right, when it is the ſhorteſt diſtance between two points; or curved, when it is not the ſhorteſt diſtance between two points.

II. A *ſuperficies* is a figure which hath length and breadth, and is incloſed or contained between right or curved lines.

Note, One curved line may contain a ſpace or ſuperficies; but of right lines, leſs than three cannot contain a ſpace.

III. When one line is inclined towards an other line, in ſuch a manner, as if either or both were continued, they would meet; then the opening of theſe lines is called *an Angle*, *Fig.* 1. *Pl.* 1.

IV. When one line ſtands ſo on another, as to incline to neither ſide; but makes the angles on each ſide equal; each of thoſe angles is called *a right one*; and the line ſo ſtanding on the other, is called *a perpendicular*, to that whereon it ſtands. *Fig.* 2.

V. All three ſided figures are called *triangles*, but admit of the following diſtinctions:

Firſt, If the three ſides are unequal, it is called *a ſcalene triangle.* *Fig.* 3.

Secondly, If the three ſides are equal, it is called *an equilateral triangle.* *Fig.* 4.

Thirdly, If only two ſides are equal, it is called *an Iſoſceles triangle.* *Fig.* 5.

Fourthlp, If it has one right angle, it is called *a right-angled triangle.* *Fig.* 6.

VI. All four ſided figures are called *quadrilaterals*, but admit of the following diſtinctions:

Firſt, When the four ſides are equal; if the angles are right ones, it is called *a ſquare*; *fig.* 7. but if the angles are not right ones, it is called *a rhombus.* *Fig.* 8.

Secondly, When the oppoſite ſides only are equal; if the angles are right ones, it is called *a rectangle*; *fig.* 9. but if the angles are not right ones, it is called *a rhomboides.* *Fig.* 10.

Note, Theſe four are called *parallelograms*, as having their oppoſite ſides parallel or equidiſtant to each other; but all other four ſided figures are called *trapeziums.* *Fig.* 11.

VII. A circle is a plane figure, bounded by one curved line called the *circumference*; to which all right lines drawn from a certain point within the figure, called *its center*, are equal. *Fig.* 12.

VIII. The *diameter* of a circle is a right line drawn through the center, terminated at each end by the circumference, and divides the circle into the two equal parts, each called *a ſemicircle.* Half the diameter is called *a radius.* *Fig.* 12.

IX. Every circumference is ſuppoſed to be divided into 360 equal parts called *degrees*; each degree into 60 equal parts called *minutes*; each minute 60 equal parts called *ſeconds*, &c. And any part of a circumference is called an *arc.*

X. The *chord* of an arc, is a right line joining the ends of an arc: Or, a right line dividing a circle into two unequal parts called *ſegments*, is called a *chord.* *Fig.* 12.

XI. If a chord cut a diameter at right angles, that part of the diameter lying between the chord and circumference, is called *a verſed ſine*, and is the height of the ſegment. *Fig.* 12.

XII. A ſector is a figure contained under two radius's of a circle, and the arc included between thoſe radius's. *Fig.* 12.

XIII. A polygon is a figure contained under many ſides; if the ſides, and angles, are equal among themſelves, the figure is called a *regular polygon*; otherwiſe, an *irregular one.*

Note, a polygon is named according to its number of ſides, *viz.* If it has 5 ſides, it is called *a pentagon*; if 6 ſides, *a hexagon*; if 7, an *heptagon*; if 8, *an octagon*; if 9, *a nonagon*; if 10, *a decagon*; if 11, *an undecagon*; if 12, *a duodecagon.* See *Fig.* 13, 14, 15, 16, 17, 18, 19, 20.

XIV. In any quadrilateral, if a line be drawn to any two oppoſite angles, that line is called *a diagonal.* *Fig.* 21.

XV. The altitude or height of any figure, is a perpendicular, let fall from the vertex of the figure to its baſe; that is, the line on which the figure is ſuppoſed to ſtand.

XVI. The area of any figure, is the ſuperficial content thereof.

SECTION II.

As there is ſometimes a neceſſity of letting fall a perpendicular, in order to come at the area of a figure; it is therefore convenient to know how to ſolve the following problems.

PROBLEM. I.

To biſect, or divide into two equal parts, the line AB. (Fig. 22.)

CONSTRUCTION.

Set one foot of the compaſſes on the end B, open the other to any convenient diſtance greater than half AB, with that opening deſcribe the arch DE; ſet one foot on A, and with the ſame opening croſs the former arch in D and E; draw the line DE, and it will biſect the given line AB in the point C.

PROBLEM II.

On any point C *of a given line* AB, *to erect a perpendicular. (Fig. 23.)*

CONSTRUCTION.

On any convenient point as D, out of the given line, ſet one foot of the compaſſes, extend the other to the point C, with that extent, deſcribe a circumference cutting AB in E; draw the diameter EDF; thro' the point C, and the extremity F of the diameter EF, draw the line CF, which will be perpendicular to the given line AB, and ſtand on the point C, as was required.

PROBLEM III.

To let fall a perpendicular to any given line AB, *from a given point* C, *above that line.* (*Fig.* 24.)

CONSTRUCTION.

Set one foot of the compaſſes on the point C, and with any convenient opening deſcribe the arch DE; on the points D and E, with the ſame opening, deſcribe arches below the line, to croſs each other in F; lay a ruler by C and F, and draw the line CF, which will be perpendicular to the line AB, as was required.

SECTION III.

THE area of all right lin'd figures, may be obtained by the help of the two following propoſitions.

PROPOSITION I.

The length and breadth of a parallelogram being known; to find its area or ſuperficial content.

RULE.

RULE.

Multiply the length by the breadth, the product will be the meaſure of the area required.

PROPOSITION II.

Having given the baſe and perpendicular height of any right lined triangle; to find the area.

RULE.

Multiply the baſe by half the perpendicular height; or the perpendicular height by half the baſe; and the product is the area.

By the help of theſe two propoſitions, workmen compute the areas of all right lined figures; and that by three different ways, *viz.*

Firſt. By *aliquot parts.*
Secondly. By *decimals.*
Thirdly. By *duodecimals.*

But the latter of theſe are moſtly uſed when the dimenſions are taken in feet, inches, *&c.*

Different kind of works are computed by different meaſures, *viz.*

Firſt. By the foot; as *glazing.*

Secondly. By the yard; as *painting*, *plaiſtering*, *paving*, *&c.*

Thirdly. By the ſquare of 100 feet; as *flooring*, *partitioning*, *roofing*, *tyling*, *&c.*

Fourthly. By the rod of 16½ feet, whoſe ſquare is 272¼ by which *bricklayers* compute their work.

Land is beſt meaſured by the *Gunter's* chain, which is 4 poles long, and is divided into 100 links, each being 7,92 inches; and as 40 poles in length, and 4 in breadth make a ſtatute acre; therefore

625 ſqu. links = 1 pole
100000 - - = 160 = 1 acre =4840 yards.

SECTION IV.

Of ARTIFICERS WORKS.

QUESTION I.

WHAT is the area or ſuperficial content of a window, whoſe length is 14 feet 6 inches, and breadth 4 feet 9 inches?

This example is here exemplify'd by the three methods of operation.

In.		*First.* By aliquot parts.	*Secondly.* By decimals.	*Thirdly.* By duodecimals.
6	½	14 - 6	14,5	14 - 6
		4	4,75	4 - 9
		58 - 0	725	58 - 0
	½	7 - 3	1015	10 - 10 - 6
		3 - 7½	580	68 - 10 - 6
		68 - 10½	68,875	

The reſult, in each of theſe methods, is the ſame, which is, 68 ſquare feet, and 10½ primes.

Altho' it is obvious, that linear feet drawn into linear feet, produce ſquare feet; and inches by inches, produce ſquare inches; and parts by parts, produce ſquare parts: Yet feet, inches and parts, drawn into feet, inches and parts, produce other figures and names beſide ſquare feet, ſquare inches, and ſquare parts; *viz.* one name or figure between the ſquare feet and ſquare inches, this multiplied by 12, give ſquare inches: another name, between the ſquare inches and ſquare parts, this multiplied by 12, gives ſquare parts. See the following example.

$$\begin{array}{rcrcrcrcrcl}
\text{f.} & & \text{i.} & & \text{p.} & & & & & & \\
39 & \cdot & 10 & . & 7 & \cdot & & & & = & A \\
18 & . & 8 & . & 4 & . & & & & = & B \\
\hline
1 & . & 1 & . & 3 & \cdot & 6 & . & 4 & = & A \times 4\text{p.} \\
26 & . & 7 & . & 0 & . & 8 & . & - & = & A \times 8\text{i.} \\
0 & . & 10 & . & 6 & . & - & . & - & = & 18\text{f.} \times 7\text{p.} \\
15 & . & 0 & . & - & . & - & . & - & = & 18\text{f.} \times 10\text{i.} \\
702 & . & - & . & - & . & - & . & - & = & 39\text{f.} \times 18\text{f.} \\
\hline
745 & \cdot & 6 & . & 10 & . & 2 & . & 4 & = & A \times B.
\end{array}$$

Or. 745 . 72 + 10 . 24 + 4.
That is 745 ſq. f. 82 ſq. i. 28 ſq. parts.

The names already given to the different denominations ariſing in duodecimal multiplication, are feet, primes, ſeconds, thirds, fourths, *&c.*; but it will not be improper in this place, to ſhew what figures thoſe names really repreſent in ſuperficial meaſure, produced by feet, inches and parts, drawn into feet, inches and parts.

1. The feet, are ſquare feet.
2. The primes, are rectangles of a foot long and an inch wide.
3. The ſeconds, are either rectangles of a foot long, and 1 part wide; or, which is juſt the ſame, they are ſquare inches.
4. The thirds, are rectangles of an inch long and 1 part wide.
5. The fourths, are ſquare parts.

I. Of meaſurements by the foot ſquare; as glazing, and maſons flat work.

QUESTION II.

What will the glazing a triangular ſky-light come to, at 10 d. ⅌ foot; ſuppoſing the baſe 12 feet 6 inches long, and the perpendicular height 6 feet 9 inches?

By duodecimals.

```
                          f.   i.
                         16 - 9 -    = the height.
                              6 - 3  = ½ the baſe.
                       ---------------
                          4 - 2 - 3
 d.   d.                100 - 6
                       ---------------
 6     4 |½, ⅓|         104 - 8 - 3  = the area.
                       ---------------
                         52
In.   d.                 34 - 8
 6|½|10                   0 - 6¾
 2|⅓| 5                ---------------
      1¾             20) 8,7 - 2¾           Anſwer,
  -----                ---------------
      6¾                  4 7 2¾            4 l. 7 s. 2¾ d.
                       ---------------
```

By decimals.

$$\begin{array}{rl}
16{,}75 & \\
\underline{6{,}25} & \\
8375 & \\
3350 & d. \quad £. \\
\underline{10050} & 10 = 0{,}041\dot{6}. \\
104{,}6875 & \\
\underline{\dot{6}\ 140{,}0,} & \text{the multiplier inverted.} \\
\underline{9)\ 628} & \\
69\dot{7} & \\
1047 & \\
\underline{41875} & \\
\underline{4{,}3619\dot{7}} & £.
\end{array}$$

In the decimal work of this example, the multiplicand 104,6875, is firſt multiplied by $\dot{6}$, and then divided by 9, (⅌ rule p. 17.) becauſe the 3 lines of product may ſtand together. And the ſame practice is to be underſtood in many of the following examples.

The area in the duodecimal work, is valued by aliquot parts.

QUESTION III.

There is a houſe with three tier of windows, three in a tier; the height of the firſt tier is 7 *feet*, 10 *inches*; *of the ſecond*, 6 *feet*, 8 *inches*; *of the third*, 5 *feet*, 4 *inches*; *and the breadth of each*, 3 *feet* 11 *inches*; *what will the glazing come to at* 14 *d.* ⅌ *foot?*

This queſtion is beſt ſolved, by adding together the heights of the windows over each other, and multiplying the ſum by 3, the number of rows; this gives a length equal to the 9 windows together.

By duodecimals.

Add $\left\{\begin{array}{r} 7 - 10 \\ 6 - \ \,8 \\ 5 - \ \,4 \end{array}\right\}$ the heights.

19 - 10

Multiply by 3 heights.

59 - 6 = heights toget.

Mult. 3 - 11 = breadth.

54 - 6 - 6

178 - 6

P. *d.* *d.* 233 - 0 - 6 = area.

6 | $\frac{1}{24}$ | 14 | 0$\frac{1}{2}$

| 2 | $\frac{1}{6}$ | 233

38 - 10

$\frac{1}{2}$

20) 27,1 - 10$\frac{1}{2}$

£. 13 - 11 - 10$\frac{1}{2}$ Anſwer.

By decimals. 59,5 = heights together.

$3,91\acute{6}$ = breadth.

9) 3570

$396\acute{6}$

595

5355

1785

d. £.

$233,041\acute{6}$ and 14 = $0,058\acute{3}$

$\acute{3}850,0$

9) 699

776

18643

116521

£. 13,5940

Glaziers generally meaſure their work to a quarter of an inch; and never make any allowances for round or oval windows, but always meaſure them to the greateſt length; for there is more trouble in cutting the glaſs to thoſe ſhapes, than the value of the glaſs omitted.

QUESTION IV.

What is a marble ſlab worth, whoſe length is 5 feet 7 inches; and breadth 1 foot 10 inches, at 6 s. ℘ foot?

		By duodecimals.	By decimals.
		5 - 7 - 0	5,583
		1 - 10	1,83
		4 - 7 - 10	9) 16750
		s. 5 - 7	18611
		5 │ ¼ │ 10 - 2 - 10	44666
		1 │ ⅕ │ 2 - 10	55833
i.p.	s.	10	10,2361
2.0 │ ⅙	6	1 - 5	0,3 = 6 s.
0.6 │ ¼	1-0	3 - 01 - 5	3,07083
0.3 │ ½	3	Anſwer.	
0.1 │ ⅓	1½	3 *l.* 1 *s.* 5 *d.*	
	½		
	1-5		

II. Of meaſurements by the yard ſquare: as paviors, painters, plaiſterers and joiners.

In theſe works, the dimenſions are taken in feet; and the reſult given in ſquare yards, each of 9 ſquare feet.

Hence divide the area found in ſquare feet by 9, the quotient will be the number of ſquare yards required.

QUESTION V.

What will the paving a court of a rectangular form come to, at 3*s.* 2*d.* ⅌ *yard; ſuppoſing the length* 27 *feet,* 10 *inches, and the breadth* 14 *feet,* 9 *inches.*

By duodecimals.

27 - 10
14 - 9
20 - 10 - 6
11 - 8
378
9) 410 - 6 - 6 s. d.
s.d. 45 - 5 - 6 - 6 at 3 - 2

2.0 | $\frac{1}{10}$ | 45
1.0 | $\frac{1}{2}$ | 3 - 10
0.2 | $\frac{1}{6}$ | 2 - 5
7 - 6
1 - 10$\frac{3}{4}$
£. 7 - 4 - 4$\frac{1}{4}$ Anſwer.

f. i. p. s. d.
3.0.0 | $\frac{1}{3}$ | 3 - 2
1.0.0 | $\frac{1}{3}$ | 1 - 0$\frac{1}{2}$
1.0.0 | $\frac{1}{1}$ | 4
0.6.0 | $\frac{1}{2}$ | 4$\frac{1}{4}$
0.0.6 | $\frac{1}{12}$ | 2
0
1 - 10$\frac{3}{4}$

By decimals. 27,83

14,75

13916

194833

1113333

2783333

9) 410,5416

45,615740 at 3.2 = 0,1583 s.d. £.

38 51,0

9) 137

152

3649

22808

45616

£. 7,2225

QUESTION VI.

One has paved a rectangular court-yard 42 feet, 9 inches in front; and 68 feet, 6 inches in depth: And in this he laid a foot-way the depth of the court; of 5 feet, 6 inches in breadth: The foot-way is laid with purbeck ſtone, at 3 s. 6 d. ℔ yard, and the reſt with pebbles, at 3 s. ℔ yard; what will the whole come to?

Find the value of the whole Court at 3 *s.* and the foot-way at 6 *d.* theſe values added together, will give the whole coſt.

```
   68 -  6        | s.  |      |
        42 - 9    | 2   | 1/19 |
   ------------   |     |      |
   51 -  4 - 6    | 1   | 1/2  |
   21 -  0        |     |      |
  136             |     |      |
  272             |     |      |
  -------------   |     |      |
9) 2928 - 4 - 6   | d.  |      |
  --------------  |     |      |
   325 - 3 - 4 - 6| 6   | 1/40 |
  The court's area.
  ----------------
   68 -  6
         5 - 6
  ---------------
   34 -  3 - 0
  342 -  6
  ---------------
9) 376 - 9
  ---------------
   41 -  7 - 9
  The foot-way's area.
```

```
 yds.   f.    i.   p.     s.
 325 -  3 -   4 -  6  at  3
 ----------------------
  32 - 10
  16 -  5
         1 -  1¼
 ----------------------
£. 48 - 16 -  1¼
 ----------------------
 yds.   f.    i.   d.
  41 -  7 -   9  at 6
 ----------------------
   1 -  0 -   6
              5
 ----------------------
£.  1 - 0 -  11
```

```
 f. i. p. |     | s.  d.
 3 - -    | 1/3 | 3 - 0
          |     | -------
    4 -   | 1/9 | 1
      6   | 1/8 |      1¼
          |     | -------
          |     | 1 - 1¼
 --------------------------
 f. i.    |     |
 3 - -    | 1/3 |  - 6 0
          |     | -------
 3 - -    | 1/1 |      2
 1 - -    | 1/3 |      2
 0 9 -    |     |      0½
          |     |      0½
          |     | -------
          |     | 0 - 5
 --------------------------
```

```
                                l.     s.    d.
The whole court at 3s. is      48  —  16  —  1¼
The foot-way at 6d. is -        1  —      — 11
                               ----------------
The whole cost ——          £. 49  —  17  —  ¼
```

QUES-

QUESTION VII.

What will the plaiſtering a ceiling at 10 *d.* ⅌ *yard come to; ſuppoſing the length* 21 *feet,* 8 *inches, and the breadth* 14 *feet,* 10 *inches?*

By duodecimals.

```
                              21 -  8
                                 14 - 10
                            -------------
                              18 -  0 -  8
                               9 -  4
                             294
                            -------------
            d.      d.  9)   321 -  4 -  8          d.
            6 and 4 |1/2,1/3| 35 -  6 -  4 - 8  at 10
                    |       |-------------
                    |       | 17 -  6
                    |       | 11 -  8
                    |       |       6 3/4
                            -------------
                       20)    29 -  8 3/4
                              -------------
                               1 -  9 -  8 3/4

 f.i. |    | d.
 3.0  |1/3 | 10
      |    |------
  .0  |1/1 |  3 1/4
  .4  |1/5 |  3 1/3
      |    |    1/4
      |    |------
              6 3/4
```

Anſwer, 1 *l.* 9 *s.* 8¾ *d.*

By decimals.

14,8$\cancel{3}$
21,$\cancel{6}$
9) 8900
9888
1483
29666
9) 321,3$\cancel{8}$ £.
35,7098, *&c. at* 10*d.* = 0,041$\cancel{6}$
$\cancel{6}$140,0
9) 214
238
357
14283
1,4878

Plaiſterers works are principally of two kinds, namely, *firſt*, works lath'd and plaiſter'd, which are call'd *ceiling*. *Secondly*, works *rendered*, which is of two kinds, *viz.* upon brick walls, or between quarters in the partition between rooms.

In meaſuring *rendering* upon brick walls, there are no deductions made; but in meaſuring *rendering* between *quarters*, there is commonly a fifth part of the whole area deducted: But when *rendering* between *quarters* is whited or colour'd, there is commonly a fourth or fifth part added to the whole area, for the ſides of the *quarters* and *braces*, &c.

Note, Make proper deductions for doors, windows, *&c.*

QUESTION VIII.

There is a quantity of partitioning that meaſures 234 *feet,* 8 *inches about, and* 14 *feet,* 6 *inches high; but is rendered between quarters: The lathing and plaiſtering will be* 8*d.* ⅌ *yard,* and the whiting 2*d.* ⅌ *yard; what will the whole come to?*

By duodecimals.		By decimals.
234 - 8		234,6
14 - 6		14,5
117 - 4 - 0		11733
9 - 4		93866
3276		234666
9) 3402 - 8		9) 3402,6
5) 378 - 0 - 8	the area in yards.	5) 378,0740
75 - 5 - 6	this $\frac{1}{5}$ part deduct.	75,6148
302 - 4 - 2	rem. the ar. plaiſter'd.	302,4592
378 - 0 - 8	to the area in yards.	378,0740
75 - 5 - 6	add the $\frac{1}{5}$ Part	75,6148
453 - 6 - 2	gives the ar. whited.	453,6888

302,4592
8 *d.* = ,03 £.
10,081273

453,6888
2 *d.* = ,0083 £.
15 12296
3629 5104
3,64463336

10,081973
3,64463336
£. 13,72660636

Anſwer, 13 *l.* 14 *s.* 6 *d.* the whole coſt.

In theſe two operations, inſtead of multiplying by 3, and dividing by 9, (as directed p. 17.) take $\frac{1}{3}$ of the multiplicand, which is exactly the ſame, but more expeditious.

QUESTION IX.

Suppoſe a room that was painted at 8 d. ℔ yard, meaſures as follows: The height, (taking in the cornice and mouldings) is 11 feet, 7 inches; the girt or compaſs, 74 feet, 10 inches; the door 7 feet, 6 inches, by 3 feet, 9 inches; five window-ſhutters, each 6 feet, 8 inches, by 3 feet, 4 inches; the breaks in the windows, 14 inches deep, and 8 feet high; the chimney 6 feet, 9 inches, by 5 feet; a cloſet the height of the room, 3½ feet deep, and 4¼ feet in front, with ſhelving, at 22 feet, 6 inches, by 10 inches; the ſhutter, door, and ſhelves, are coloured on both ſides: What will the whole come to?

$74{,}8\cancel{7}\times 11{,}58\cancel{7} =$	866,8194	The area of the whole room.
$6{,}\cancel{6}\times 3{,}\cancel{7}\times 5 =$	$111{,}\cancel{1}111$	The area of the ſhutters once.
$(\overline{8+8+3{,}\cancel{7}+3{,}\cancel{7}}=)\,22{,}\cancel{6}\times 1{,}16\times 5 =$	$132{,}\cancel{1}222$	The breaks in the windows.
$7{,}5\times 3{,}75 =$	28,125	The door once.
$(\overline{\overline{3{,}5\times 2}+\overline{4{,}75\times 2}}=)\,16{,}5\times 11{,}583 =$	191,125	The cloſet's area.
$22{,}5\times {,}8\cancel{7}\times 2 =$	37,5	The area of the Shelves.
The ſum of the areas.	$1366{,}902\cancel{7}$	Out of which deduct.
$6{,}75\times 5 =$	33,75	The chimney's area.
Remains	$1333{,}152\cancel{7}$	The area of the whole work.

Then $\frac{1333{,}152\cancel{7}}{9} = 148{,}128$, *&c. yds.* then if 1 *yd.* : ,03 £. : : 148,128 *yd.* : 4,9376 £. *&c.* = 4 £. 18 *s.* 9 *d.*

Painters take their dimenſions with a ſtring, and meaſure from the top of the cornice to the floor, girting the ſtring over all the mouldings and ſwelling pannels: and in meaſuring of doors, they account the height and breadth of the door ſo much more, as in the thickneſs of the ſtuff; it being reaſonable they ſhould be paid for all places whereon their colour is laid. Their price they generally proportion according to the number of times they lay their colour on.

Note, There muſt always be made deductions for Chimnies, Caſements, &*c.* if any within the dimenſions taken.

QUESTION X.

What will the wainſcoting a room come to at 6 *s. per ſquare yard; ſuppoſing the height of the room (taking in the cornice and mouldings) is* 12 *feet* 6 *inches, and the compaſs is* 83 *feet* 8 *inches; the window ſhutters each* 7 *feet* 8 *inches, by* 3 *feet* 6 *inches, and the door* 7 *feet by* 3 *feet* 6 *inches; the ſhutters and door being worked on both ſides, is reckoned work and half work?*

83 - 8
12 - 6

41 - 10 - 0
1004 - 0

1045 - 10 - 0 = { room's area, including the ſhut. and do. once.

7 - 8
3 - 6

3 - 10 - 0
23 - 0

26 - 10 - 0
3

80 - 6 - 0 = three ſhutters once.

3 - 6
7 - 0

24 - 6 - 0 = door once.

2) 105 - 0 = ſhutters and door.

52 - 6 = half the ſhutters and door.

		f.	*i.*	
		1045 -	10 =	room
		52 -	6 =	$\frac{1}{2}$ ſhut. and door.
	9)	1098 -	4 =	whole work.

	s. d.		*yds.*	*f.*	*i.*	
	4.0	$\frac{1}{3}$	122 -	0 -	4	at 6 *s.* ℔ yard.
			l.	*s.*	*d.*	
in.	2.0	$\frac{1}{2}$	24 -	8 -	0	
4 $\frac{1}{27}$	6.0		12 -	4 -	0	
	0.2$\frac{1}{2}$			0 -	2$\frac{1}{2}$	
			£. 36 -	12 -	2$\frac{1}{2}$	the anſwer.

Joiners meaſure their work in height with a ſtring, and their length or compaſs upon the floor as painters do; for they ſay, they ought to meaſure where their plane touches, therefore they take the cornice and mouldings into the height of the room.

Note, They only take the cornice and mouldings in with the height of the room, when they are ſtruck with a *horſe plane*, (as they call it) but if they are wrought by hand, then they are paid ſo much ℔ *foot* running meaſure.

All ſtuff an inch and half thick and under, wrought on both ſides, is by them reckoned at work and half work; but ſtuff of greater thickneſs wronght on both ſides, is valued at double work.

They make deduction for all vacancies that fall wihin their work; and *window boards*, *ſofite boards*, *cheeks*, *&c.* are meaſured by themſelves.

III. Of meaſurements by the ſquare. As flooring, partitioning, roofing, tyling, *&c.*

In theſe works, the dimenſions are taken by a rod of ten feet; and therefore the reſult is in ſquares of 100 ſquare feet each.

Hence, divide the area found in ſquare feet by 100, the quotient will be the number of ſquares required.

QUESTION XI.

Suppoſe a houſe of three ſtories, beſide the ground-floor, was to be floor'd at 6*l.* 10*s.* ℔ *ſquare; the houſe meaſures* 20 *feet* 8 *inches, by* 16 *feet* 9 *inches; there are* 7 *fire-places, whoſe meaſures are; two, each of* 6 *feet, by* 4 *feet* 6 *inches; two other, each of* 6 *feet, by* 5 *feet* 4 *inches; and two, each of* 5 *feet* 8 *inches, by* 4 *feet* 8 *inches; and the ſeventh,* 5 *feet* 2 *inches, by* 4 *feet; and the well-hole for the ſtairs, is* 10 *feet* 6 *inches, by* 8 *feet* 9 *inches; what will the whole come to?* 4 — 6

4 - - 6
6
27 - - 0
2
54

5 - - 4
6
32 - - 0
2
64

5 - - 8
4 - - 8
3 - - 9 - - 4
22 - 8
26 - - 5 - - 4
2
52 - - 10 - - 8

5 - - 2
4
20 - 8

10 - 6
8 - 0
7 10 - 6
84
91 - 10 - 6
4 the number of floors.

20 - 8
16 - 0
15 - 6 - 0
330 - 8
346 - 2
4
1384 - 8 the area of the four floors.
559 - 0 8 the area of the deduct.
100) 825 - 7 - 4 the area of the work.

ſ. f. i. p. £. s.
8 - 25 - 7 - - 4 at 6 - - 10 per ſquare.

l. s. d
Anſwer, 53 - - 13 - - 3¼

367 - 6 - 0 the well hole.
54 - 0 - 0 the firſt chimney.
64 - 0 - 0 the ſecond chimney.
52 - 10 - 8 the third chimney.
20 - 8 - 0 the fourth chimney.
559 - 0 - 8 the whole deductions.

s.
10 - ½
qrs.
8
6
48
4
1 - 13 - 3¼
£. 53 - 13 - 2¼

		l. s. d.
		6 - 10 - 0
25 f.	¼	1 - 12 - 6
6 i.	$\frac{1}{50}$	7¾
1	$\frac{1}{6}$	1¼
4 p.	$\frac{1}{3}$	¼
		1 - 13 - 3¼

In flooring they deduct the hearth-ſtone, except it has a border round it; and then the hearth is meaſured in with the floor.

QUESTION XII.

In 173 *feet* 10 *inches in length, and* 10 *feet* 7 *inchss in height of partioning; how many ſquares?*

By duodecimals.

173 - 10
10 - 7
101 - 4 - 10
1738 - 4
100) 1839 - 8 - 10
ſ. f. i. p.
18 — 39 — 8 — 10

By decimals.

$1\ 7\ 3{,}8\ \cancel{3}$
$\cancel{3}\ 8\ 5{,}0\ 1$ } breadth inverted.
9) 5 2
5 8
1 3 9 0
8 6 9 1
1 7 3 8 3 3 ſqrs.
100) 1 8 3 9,7 2 (1 8,3 9 7 2

In *roofing*, *tyling* and *ſlating*, it is cuſtomary to reckon the flat and half of any building within the walls, to be the meaſure of the roof of that building; when the ſaid roof is of a *true pitch*.

Note, All roofs are ſaid to be of a *true pitch*, when the rafters are $\frac{3}{4}$ of the breadth of the building.

If the roof is more or leſs than *true pitch*, they meaſure from one ſide to the other with a rod or ſtring.

QUESTION XIII.

If a houſe meaſure within the walls 52 *feet,* 8 *inches in length, and* 30 *feet,* 6 *inches in breadth; and the roof be of a true pitch; what will it coſt roofing at* 10 *s.* 6 *d.* ℔ *ſquare?*

```
By duodecimals.                                                              By decimals.
       30 - - 6 — the breadth of the building ——                                 30,5
       15 - - 3 —— the half breadth —— ——                                        15,25
       45 - - 9 —— the breadth of the roof —— ——                                 45,75
            52 - - 8                                                             52,6
       30 - - 6 - - 0                                                       9) 27450
       39 - - 0                                                                30500
       90                                                                      9150
      255                                                                     22875
  100) 2409 - - 6                                                        100) 2409,5
s. d.   | sq.  f.   i.     s.  d.                                           24,095 ſqrs.
10.0 ½  | 24 - 9 -  6 at 10 - 6 per ſquare.                                   ,525
0.6 1/20| 12 -  0          | f.     f.  |           | s.   d.               120475
        | - - -  12        | 5 and 4 1/20, 1/25     | 10 -  6               48190
        | - -  - - - 11¾   |       1    |           | — 6¼               120475
        | 12 - 12 - 11¾    |       6    | 1/8       | — 5                12,649875
                                                    | — ½
                                                    | — 11¼          Anſw. 12 l. 12 s. 11¾ d.
```

There are other works about a building done by the carpenter, that are meaſured by the foot *running meaſure*; as *cornices*, *doors*, *and caſes*; *window-frames*, *lintels*, *guttering*, *ſommers*, *ſkirt-boards*, *&c.*

In the meaſuring of roofing for workmanſhip alone; they generally deduct the holes for chimny ſhafts and ſkylights, if they are any thing conſiderable.

But meaſuring for work and materials, they commonly meaſure in all *ſkylights*, *lutheren lights*, and *holes* for the *chimney ſhafts*, for their trouble and waſte of *ſtuff*; excepting ſuch *ſky-lights* as exceed nine or ten feet in area.

QUESTION XIV.

What will the tyling a barn coſt, at 25 *s.* 6 *d.* ⅌ *ſquare*; *the length being* 43 *feet*, 10 *inches*, *and breadth* 27 *feet*, 5 *inches on the flat*; *the eaves boards projecting* 16 *inches on each ſide?*

By duodecimals.

27 - 5 —— the breadth ——

13 - 8 - 6 — the half breadth ——

1 - 4 × 2 = 2 - 8 —— the eaves boards doubled —

43 - 9 - 6

43 10 - 0

36 - 5 - 11 - 0

1 - 9 - 6 s. d. 25 - 6 = 1,275

32 - 3

129

172

100) 1919 - 6 5

f. f. i. p. s. d.

19 - 19 - 6 - 5 at 25 - 6 *per ſquare.*

s. d. | f. f. | s. d.

5 | $\frac{1}{4}$ | 19 | 10 4 | $\frac{1}{10}$ $\frac{1}{25}$ | 25 - 6

0 6 | $\frac{1}{10}$ | 4 - 15 | 5 | $\frac{1}{2}$ | 2 0$\frac{1}{2}$

9 - 6 | 6 i. | 1 - 3$\frac{1}{4}$

4 - 11$\frac{1}{2}$ | $\frac{1}{8}$ | 1 - 0$\frac{1}{4}$

Anſ. *l.* 24 - 09 - 05$\frac{1}{2}$ | 1$\frac{1}{2}$

4 - 11$\frac{1}{2}$

By decimals.

2 7,4 1 $\cancel{6}$ 6

1 3,7 0 8 $\cancel{3}$

2,$\cancel{6}$ 6 6 0

4 3,7 9 1 $\cancel{6}$

• $\cancel{3}$ 8,3 4 length inverted.

9) 1 3 1 4

1 4 6 0

3 5 0 3 3

1 3 1 3 7 5

1 7 5 1 6 6 6

100) 1 9 1 9,5 3 4 (1 9,1 9 5 3 4 *ſqrs.*

5 7 2,1

1 9 1 9 5 3

3 8 3 9 0

1 3 4 3 7

9 6 0

2 4.4 - 4

Note, When the ridge of a roof forms an angle, then the tiles coming from the ridge to the eaves make an angle alſo; that angle of the roof, which bends inwards, is alled a *valley*; and the angle bending outwards, is called a *Hip*; and in tiling and ſlating, it is common to add the length of the *Vallies* (meaſured from the ridge to the eaves) to the content in feet; ſometimes the *Hips* are added.

In ſlating, it is common to reckon the breadth of the roof 2 or 3 inches broader than what it meaſures, becauſe the firſt row is almoſt cover'd by the ſecond; this is done ſometimes when a roof is tiled.

Note, Skylights and chimney ſhafts are deducted, but they ſeldom deduct luthern lights (or garret windows on the roof) for the covering of ſuch, they reckon it equal to the hole in the roof.

IV. Of meaſurements by the rod, as brickwork

In this work, the dimenſions are eſtimated by a rod of 16½ feet: and therefore the reſult is in ſquare rods of 272¼ ſquare feet each.

Hence (for practice) divide the area found in ſquare feet by 272, the quotient will be the rods required.

Note, Bricklayers always compute or value their work at the rate of a brick and an half thick; and if the thickneſs of the wall happen to be more or leſs than ſuch, it muſt be reduced to that thickneſs as follows:

Multiply the area of the wall, by the number of half bricks the thickneſs of the wall is of; divide the product by 3, and it gives the area.

QUESTION XV.

How many ſquare rods are there in a wall 62½ *feet long,* 14 *feet,* 8 *inches high, and* 2½ *bricks thick?*

By duodecimals.

62 - 6
14 - 8
41 - 8 - 0
875 - 0
r. *f.* *i.*
272) 916 - 8 (3 - 100 - 8
5
3) 16 - 231 - 4
5 - 167 - 9 - 4

r. *f.* *i.* *p.*
Anſw. 5 - 167 - 9 - 4

By decimals.

62,5
14,6
9) 3750
4166
8750
272) 916,6 (3,37, *&c. rods.*
5
3) 16,85
5,616, *&c.*

In meaſuring of brickwork, they are very careful in regard to their allowances; for one foot ſquare in the front is commonly worth ſixpence.

In great buildings, they often deduct the timbers laid in the walls; but this is only when the workmanſhip is very good; for in general, it is allow'd in; becauſe they reckon the time they wait on the carpenter, together with the bedding of thoſe timbers in mortar, is equal to the brickwork that would ſupply the timbers place.

QUESTION XVI.

Suppoſe the ſide walls of an houſe to be 28 feet, 10 inches in length; and the height of the roof from the ground, 53 feet, 8 inches; and the gable (or triangular part at top) to riſe 42 courſe of bricks (reckoning 4 courſe to a foot). Now 20 feet high is $2\frac{1}{2}$ bricks thick; 20 feet more, at 2 bricks thick; 15 feet 8 inches more, at $1\frac{1}{2}$ brick thick; and the gable of 1 brick thick; what will the whole work come to at 5 l. 16 s. ℘ rod?

$\frac{42}{4} = 10,5$ the height of the gable; its half is 5,25 *feet*.

28,88	28,88	28,88	28,88
20	20	15,6	5,25
576,6	576,6	9) 17300	14416
5	4	1922	57666
3) 28833	3) 23066	43250	1441666
961,1	768,8	451,72	151,375
f.	*f.*	*f.*	2
			3) 302,75

As 272 ; 5,8 : : 2282,638 : 48,671
(*f.* £. *f.* £.)

100,966 at 1
451,722 at $1\frac{1}{2}$
768,888 at 2
961,111 at $2\frac{1}{2}$
} Bricks thick.
2282,638

£. s. d.
Anſwer, 48 - - 13 - - $5\frac{1}{4}$.

In all buildings, the thickneſs of the walls generally decreaſe as they riſe; and it is uſual to ſet off half a brick at each decreaſe: The thickneſs is commonly ſet off on the inſide, and that in a place where a floor will come, ſo that the ſet-off is thereby hid.

It is common to build from a baſe 4 courſe of bricks high, and which projects two or three inches on each ſide of the wall.

The different thickneſſes are meaſured ſeparate, and reduced each to a brick and an half thick; and then added together.

To meaſure a chimney ſtanding by itſelf, without any party-wall being adjoined; girt it about for the length, and reckon the height of the ſtory for the breadth: The thickneſs muſt be the ſame the Jaumbs are off, provided that the chimney be wrought upright from the mantle-tree to the ceiling; not deducting any thing for the vacancy between the floor (or hearth) and the mantle-tree; becauſe of the gathering of the breaſt and wings, to make room for the hearth in the next ſtory.

If the chimney-back be a party-wall, and the wall be meaſured by itſelf; take the depth of the two Jaumbs, and the length of the breaſt for a length; and the height of the ſtory is the breadth, at the ſame thickneſs your Jaumbs were off.

To meaſure chimney ſhafts, or that part which appears above the roof; girt them with a line, about the leaſt place for the length; and take the height for the breadth; and if they be four inches thick, ſet down the thickneſs at one brickwork; but if they be wrought nine inches thick, (as ſometimes they are, when they ſtand alone and high, above the roof) reckon the thickneſs at a brick and half, in conſideration of *wyth* and *pargetting*, and trouble of ſcaffolding.

It is cuſtomary in moſt places to allow double meaſure for chimnies.

More EXAMPLES, *to exerciſe the foregoing* PROPOSITIONS.

N. B. The areas of parallelograms and triangles, being divided by one of their dimenſions, will give the other dimenſion.

QUESTION XVII.

What difference is there between a floor 48 *feet long, and* 30 *feet broad; and two others, each of half the dimenſions?*

Now $48 \times 30 = 1440$; and $24 \times 15 \times 2 = 720$; but 720 is half of 1440.

Therefore any plane figure, whoſe linear dimenſions are double to the dimenſions of another like figure, contains four times the area.

QUESTION XVIII.

From a mahogany plank 26 *inches broad; a yard and an half in area is to be ſaw'd off; what diſtance from the end muſt the line be ſtruck?*

Now $1\frac{1}{2}$ yards area $= 13\frac{1}{2}$ feet area.
And 26 inches $= 2,1\not{6}$ feet.

Then $\frac{13,5}{2,1\cancel{6}} = 6,23$ feet, the diſtance from the end the line muſt be ſtruck.

QUESTION XIX.

A joiſt is 8½ *inches deep, and* 3½ *broad; I want a ſcantling juſt as big again, that ſhall be* 4¾ *inches broad; what will be the other dimenſion?*

Now $8,5 \times 3,5 \times 2 = 59,5$; and $\frac{59,5}{4,75} =$ 12,52 inches deep, the anſwer.

QUESTION XX.

I have a girder 19 *inches by* 13; *but one that has but a quarter of the timber in it, ſo it be* 10 *inches deep will ſerve my purpoſe; how broad muſt it be?*

Now $19 \times 13 = 247$, and $\frac{247}{4} = 61,75$.

Then $\frac{61,75}{10} = 6,175$ inches, the breadth.

QUESTION XXI.

A roof is 24 *feet,* 8 *inches, by* 14 *feet* 6 *inches on the flat; and covered with lead at* 8 ℔ *to the foot; what will it come to at* 18 *s.* ℘ *Cwt.*

Now 24,6 × 14,5 = 357,6 f. the area.

And 8 ℔ = 0,071428 *Cwt.*

Then as 1 f. : 0,071428 *Cwt.* : : 357,6 f. : 25,5473 *Cwt.*

Then as 1 *Cwt.* : 0,9 £. : : 25,5473 *Cwt.* : 22,992 £.

Or 22 £. 19 *s.* 10¼ *d.* is the coſt.

QUESTION XXII.

A plumber made for a maſon a leaden ciſtern, every foot ſquare whereof weigh'd 19 ℔ *at* 19 *s.* ℘ *Cwt. the dimenſions were* 81 *inches long,* 46 *inches deep, and* 36 *inches broad; with three ſtays acroſs it, of the ſame ſtrength, and each* 18 *inches deep; For which the maſon was to pave with purbeck ſtone, at* 7½ ℘ *foot, a ſquare pavement, that ſhould juſt balance the coſt of the ciſtern; what muſt the ſide thereof be?*

Now 81 + 36 = 117, which × by 2 gives 234 = the length of the two ſides and two ends; or the girth of the ciſtern; then 234 × 46 = 10764 inches, the area of the ſides and end: And 81 × 36 = 2916 inches, the area of the bottom.

Again, 36, the length of one ſtay, × by 18 the depth; gives 648 inches, which × 3, gives 1944 inches, the area of the three ſtays. Then 10764 + 2916 + 1944 = 15624 inches, the area of the whole ciſtern and ſtays.

Note, 19 ℔ =, 0,1696, *&c. Cwt.*

Then, as 144 *ſq. in* : ,1696 *Cwt.* : : 15624 *ſq. in.* : 18,4016 *Cwt.* The ſhorteſt way to work this proportion is, divide the third term by the firſt term, gives 108,5, and × the ſecond term by this quote, gives the fourth term, which is the weight of the ciſtern.

Then if 1 Cwt. : ,95 £. : : 18,4016 Cwt. 17,48152 £. = 17 £. 9 s. 7½ the expence that the ciſtern does amount to.

Therefore, as ,03125 £. : 1 f. : : 17,48152 £. : 559,408 f. the area of the ſquare; and extracting the ſquare root, will give 23,65 f. &c. the ſide thereof.

QUESTION. XXIII.

There is a ſleight of iron railing 42 feet long, the bars whereof are ¾ of an inch ſquare, and the whole weight is 12¾ Cwt. which is to be changed for ſome that are inch and ⅘ ſtrong, exchange at 4½ d. ꝑ ℔? what will the whole come to?

Now as the ſtrength of the bars are expreſſed by the areas of the ends.

ſtr. *ſtr.* *Cwt.* *Cwt.*

Therefore $\overline{0,75}^{2} : \overline{3,24}^{2} :: 12,75 : 67,68.$

℔ *d.* ℔ *d.*

And 1 : 4½ : : 112 : 504 = 2 £. 2 s.

Cwt. £. *Cwt.* £.

Then 1 : 2,1 : : 67,68 : 142,128.

Or the expence will be 142 £. 2 s. 7 d.

QUESTION XXIV.

A triangular field that is 1777,7 links in the baſe; and 900 links in the perpendicular; brings in 36 £. ℘ annum: How much is it let for ℘ acre?

Now $1777{,}7 \times \frac{900}{2} = 800000$ ſquare links the area.

And $\frac{800000}{100000} = 8$ acres.

Then $\left(\frac{36}{8} = 4{,}5\,£. =\right)$ 4 £. 10 s. the rent ℘ acre.

QUESTION XXV.

If a man can mow the graſs ſtanding on a ſquare rod in one minute: How long will he be in mowing a meadow whoſe length is 728 yards, and breadth 358 yards, ſuppoſing him to work 14 hours ℘ day?

Now $778 \times 358 = 260624$ yards.

Or 8615,669 ſquare rods.

Then 8615,669 min. = 10 days 3 h. 3 min.

SECTION V.

Of the areas of divers right-lined figures.

PROPOSITION III.

Given the three ſides of a triangle; to find the area.

RULE.

FROM half the ſum of the three ſides, ſubtract each ſide ſeverally; let the half ſum, and the three differences, be multiplied continually; the ſquare root of the product will be the area required.

EXAMPLE I.

If the ſides of a triangular field are 15, 14, 13, *perches; what is the content of that field.*

Now ſum of ſides $= (15 + 14 + 13 =)\ 42$;
$\frac{1}{2}$ ſum $= 21$;
And $21 - 15 = 6$; $21 - 14 = 7$; $21 - 13 = 8$:
Then $21 \times 6 \times 7 \times 8 = 7056$.
And the ſquare root of 7056 is 84 ſquare perches the area required.

EXAMPLE II.

A field of a triangular form, whoſe ſides are 380, 420 and 765 yards, letts for 55 s. ⅌ acre; how much does the whole bring in ⅌ annum?

Now $380 + 420 + 765 = 1565$.

And $\frac{1565}{2} = 782{,}5$ yards, the half ſum of the three ſides.

And $782{,}5 - 380 = 402{,}5$, the firſt difference.
$782{,}5 - 420 = 362{,}5$, the ſecond differ.
$782{,}5 - 765 = 17{,}5$ the third difference.

Alſo $782{,}5 \times 402{,}5 \times 362{,}5 \times 17{,}5 = 1998003710{,}9375$, whoſe ſquare root is 44699,034 ſquare yards, *&c.* which divided by 4840 (the ſquare yards in one acre) gives 9,2374 acres.

Then, as 1 acre : 2,75 £. : : 9,2374 acre : 25,40285 £. = 25 £. 8 *s.* 0$\frac{1}{2}$ *d.* the rent of the whole ⅌ *annum.*

PROPOSITION IV.

Two ſides of a right-angle triangle being given, to find the other ſide.

CASE I.

The two perpendicular ſides (or legs) being given, to find the other ſide, or hypothenuſe. *Fig.* 25.

RULE.

RULE.

Square each ſide, add the ſquares together, and the ſquare root of this ſum gives the *hypothenuſe* required.

CASE II.

If the *hypothenuſe* and one *leg* be given, to find the other *leg*.

RULE.

From the ſquare of the *hypothenuſe*, ſubtract the ſquare of the given *leg*; the ſquare root of the remainder, gives the *leg* required.

EXAMPLE I.

Wanted the length of a ſhoar, that ſtrutting 14 *feet from the upright of a building, may ſupport a jaumb* $20\frac{1}{2}$ *feet from the ground?*

Now 20,5 × 20,5 = 420,25; and 14 × = 196; then 420,25 + 196 = 616,25; whoſe ſquare root is 24,82 feet, *&c.* the length of the *ſhoar* required.

EXAMPLE II.

A line of 380 *feet will reach from the top of a precipice that ſtands cloſe by a brook-ſide, to the oppoſite bank: And the precipice is known to be* 128 *feet high; how broad is the brook?*

Now 380 × 380 = 144400; and 128 × 128 = 16384; then 144400 — 16384 = 128016; whoſe ſquare root is 357,79 feet, &c. the breadth of the brook required.

EXAMPLE III.

A ladder $52\frac{1}{2}$ feet long, may be ſo placed in a ſtreet, that it ſhall reach a window 29 feet from the ground on one ſide; and by turning the ladder over, (without removing the foot,) it will touch a moulding 40 feet from the ground on the other ſide; how broad is the ſtreet?

Firſt, $52,5 \times 52,5 = 2756,25$; and $29 \times 29 = 841$; then $2756,25 - 841 = 1915,25$; whoſe ſquare root is 43,76 feet, the breadth between the ladder and building the firſt ſituation.

Secondly, $40 \times 40 = 1600$; and $2756,25 - 1600 = 1156,25$; whoſe ſquare root is 34 feet, the breadth between the ladder and building, the ſecond ſituation.

Then $43,76 + 34 = 77,76$ feet, the breadth of the ſtreet required.

PROPOSITION V.

To find the area of any trapezium, the diagonal, and perpendiculars let fall thereon from the oppoſite angles, being given.

RULES.

1. Multiply the given diagonal by the ſum of the perpendiculars, and take half the product.

2. Or, Multiply the diagonal by half the ſum of the perpendiculars.

3. Or, Multiply half the diagonal by the ſum of the perpendiculars.

4. Or, Find the area of each triangle, and take their ſum.

Either of theſe rules will give the area required.

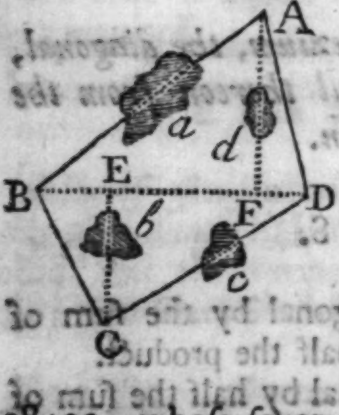
EXAMPLE I.

What is the area of a trapezium, whoſe diagonal is 34 feet, 9 inches, and the ſum of the perpendiculars is 28 feet, 6 inches?

Now half of 28 f. 6 inch. is 14 f. 3 in. = 14,25 f.
And 34,75 × 14,25 = 495,1875 feet, the area.

EXAMPLE II.

In the quadrilateral meadow ABCD, *the four ponds* a, b, c, d, *prevent the meaſuring of any other lines than the diagonal* BD, *which is* 378 *yards; the ſide* BC = 220 *yards, and the ſide* AD = 265 *yards: But it is known, that the perpendicular* CE *will fall* 100 *yards from* B; *and the perpendicular* AF *falls* 70 *yards from* D: *Required the area of that meadow?*



Firſt find the perpendiculars CE, AF, by prop. IV.
Thus 265×265=70225.
And 70 × 70 = 4900.
And 70225 − 4900 = 65325, whoſe ſquare root is 255,58, *&c* = CE.
Again, 220 × 220 = 48400.
And 100 × 100 = 10000.
Then 48400 − 10000 = 38400; whoſe ſquare root is 195,95, *&c.* = AF.

And $\frac{255,58 + 195,95}{2} = 225,76$ the half ſum of the perpendiculars.

Then $\frac{225,76 \times 378}{4840} = 17,632$ acres.

PROPOSITION VI.

To find the area of a trapezium having two ſides parallel; thoſe ſides and their perpendicular diſtance being known.

RULES.

Multiply half the ſum of the parallel ſides by the breadth.

Or, Multiply the ſum of the parallel ſides, by half the breadth.

Or, Multiply the ſum of the parallel ſides by the breadth, and take half the product.

Either of which will give the area.

EXAMPLE

What is the area of a trapezium, having two parallel ſides, the one 180 *links, the other* 120; *and whoſe breadth is* 80 *links?*

Now $180 + 120 \times \frac{80}{2} = 12000$ ſquare links.

And 12000 ſq. l. = 0 ac. 0 r. 19 p. the area.

PROPOSITION VII.

To find the area of right-lin'd irregular figures.

RULE.

Divide the ſigure into as many triangles as may be, by drawing lines from any one angle to all the other angles; and the ſum of the areas of the ſeveral triangles will be the area of the figure. *Fig.* 29.

But the lines (frequently) may be better diſpoſed, than by drawing them all from the ſame angle; *viz.* by dividing the figure into trapeziums, as many as may be, and leaving as few ſingle triangles as poſſible; for the area of a trapezium is more expeditiouſly obtained, than the areas of the two triangles which compoſe the trapezium, found ſeparately. *Fig.* 30.

Note, All right lined figures may be divided into as many triangles (without any of the dividing lines cutting each other) as the figure has ſides, abating two.

EXAMPLE.

Suppoſe a meadow, an irregular figure of 8 *ſides, which is let for* 34 *ſhillings* ꝑ *acre; upon the menſuration is divided into three trapeziums: Iu the firſt, the diagonal is* 4 *chains and* 24 *links and the ſum of the perpendiculars is* 3 *chains* 67 *links; in the ſecond, the diagonal is* 7 *chains* 43 *links, and the ſum of the perpendiculars is* 5 *chains* 38 *links; in the third, the diagonal is* 6 *chains* 78 *links, and the ſum of the perpendiculars is* 4 *chains* 84 *links; what will the whole bring in* ꝑ *annum? Fig.* 30.

Now 4 *C.* 24 *L.* = 424 *L.* and 3 *C.* 67 *L.* = 367 *L.*

Then $367 \times \frac{424}{2} = 77804$ *L.* the area of the firſt trapezium.

Alſo 7 *C.* 43 *L.* = 743 *L.* and 5 *C.* 38 *L.* = 538 *L.*

Then $743 \times \frac{538}{2} = 199867$ *L.* the area of the ſec.

Again, 6 *C.* 78 *L.* = 678 *L.* and 4 *C.* 84 *L.* = 484 *L.*

And $678 \times \frac{484}{2} = 164076$ *L.* the area of the third.

Then $77804 + 199867 + 164076 = 441747$ *L.* = 4,41747 *acres*, which at 34 *s.* = 1,7 *l.* comes to 7,509699 £. = 7 £. 10 *s.* 2¼ *d.* the anſwer.

PROPOSITION VIII.

The ſide of a regular polygon, its perpendicular diſtance from the centre, and the number of ſides being given; to find the area of that polygon.

RULE.

Multiply half the ſum of the ſides by the perpendicular.

Or, Multiply the ſum of the ſides by half the perpendicular.

Or, Multiply the ſum of the ſides by the perpendicular, and take half the product.

Either of theſe rules will give the area.

Ex-

EXAMPLE.

What is the area of a regular pentagon, whoſe ſide is 25 *yards; and the perpendicular let fall from the centre to one of its ſides, is* 17,2 *yards.*

$$\left.\begin{array}{l}\text{Then } \dfrac{25 \times 5}{2} \times 17{,}2 \\ \text{Or } 25 \times 5 \times \dfrac{17{,}2}{2} \\ \text{Or } \dfrac{25 \times 5 \times 17{,}2}{2}\end{array}\right\} = 1075 \text{ ſquare yards, is the area ſought.}$$

The ſubject of regular polygons, will be more fully treated hereafter.

SECTION VI.

Of a circle and its parts.

THERE is no figure that affords a greater number of uſeful properties than the circle; but the baſis of theſe depend on knowing the relation which the diameter has to the circumference.

The determining of this proportion has exerciſed the thoughts of ſeveral mathematicians, both ancients and moderns. Some, in expectation of diſcovering the proportion accurately, but herein they always failed; for it has been demonſtrated, that the relation of the diameter to the circumference, is not to be expreſſed in known meaſure: But others, more knowing, have contented themſelves in approximating this relation.

Archimedes's proportion is in whole numbers, *viz.* of 7 to 22; that is, ſuppoſing the diameter of a circle to be 7; he found the circumference of the ſame circle to be 22 nearly; for 22 is to much.

Metius found out a proportion of 113 to 355; that is, ſuppoſing the diameter of a circle to be 113, he found the circumference of the ſame circle, 355 nearly; but it is alſo too great, tho' much nearer the truth than *Archimedes*.

Van *Culen*'s proportion is much more accurate than any before his time; for he, ſuppoſing the diameter of a circle to be 1, found, (with prodigious labour and trouble) the circumference to be 3,141592653.5897932384.6264338327.950288 which was then thought ſo great a work, and ſo curious a performance, that the numbers were cut on his tomb-ſtone in the church-yard of St. *Peter*'s at *Leyden* (as related by ſome.)

But by methods which the moderns are poſſeſſed of, the ſame thing (and many others of a like intricate nature) may be perform'd with abundantly leſs labour and trouble; as is ſufficiently ſhewn by that excellent mathematician *William Jones*, Eſq; in his *ſynopſis palmariorum matheſeos*, where he gives Mr. *Machin*'s proportion of the diameter to the circumference, true to 100 places; that is, ſuppoſing (as Van *Culen* did) the diameter to be 1, he found the circumference to be 3,141592653.5897932384.6264338327.9502884197.1693993751.0582097494.4592307816.4062862089.9862803482.5342117067.9 + true to 100 places of figures, and that computed in very little time, compared with the methods uſed by the ancients.

The proportion of Van *Culen* is now moſt commonly uſed, and ſeems the beſt adapted to practice; as it ſaves the trouble of diviſion: but then it muſt be obſerv'd, that it is thought accurate enough, in moſt practical affairs to uſe the number 3,1416, the exceſs, being not $\frac{1}{10000}$ part of the unit.

The relation between the diameter and circumference being once eſtabliſhed, it is eaſy to deduce a great variety of uſeful properties, relating not only to the circle, but alſo, to its circumſcribing and inſcrib'd regular polygons; ſome of the moſt applicable to practical meaſuring, are contain'd in the enſuing pages; wherein, the computations are conſiderably facilitated by the application of the factors in the following tables.

A TABLE of uſeful factors, wherein p repreſents the circumference of a circle whoſe diameter is 1.

p	=	3,1415927	$p\sqrt{2}$	=	4,4428829
$2p$	=	6,2831853	$p\sqrt{\frac{1}{2}}$	=	2,2214415
$4p$	=	12,5663706	$\frac{1}{p}\sqrt{\frac{1}{2}}$	=	0,2250791
$\frac{p}{2}$	=	1,5707963	$\frac{1}{p}\sqrt{2}$	=	0,4501581
$\frac{p}{4}$	=	0,7853982	$\sqrt{p}$	=	1,7724538
$\frac{4p}{3}$	=	4,1887902	$\frac{1}{2}\sqrt{p}$	=	0,8862269
$\frac{p}{6}$	=	0,5235988	$2\sqrt{p}$	=	3,5449076
$\frac{p}{8}$	=	0,3926991	$\sqrt{\frac{p}{2}}$	=	1,25331414
$\frac{p}{12}$	=	0,2617994	$\sqrt{\frac{2}{p}}$	=	0,7978846
$\frac{p}{360}$	=	0,0087267	$\sqrt{\frac{1}{p}}$	=	0,56418957
$\frac{1}{p}$	=	0,3183099	$2\sqrt{\frac{1}{p}}$	=	1,1283791
$\frac{2}{p}$	=	0,6366197	$\frac{1}{2}\sqrt{\frac{1}{p}}$	=	0,2820948
$\frac{4}{p}$	=	1,2732395	pp	=	9,8696044
$\frac{1}{4p}$	=	0,0795775	$\frac{1}{pp}$	=	0,1013210
$\sqrt{2}$	=	1,4142136	$\frac{1}{2pp}$	=	0,0506605
$\sqrt{\frac{1}{2}}$	=	0,7071068	$\frac{1}{6pp}$	=	0,0168868

In regular polygons, whoſe number of ſides do
1ſt. The ſide of the polygon.
2d. The radius of the circumſcribing circle.
Then any one of theſe four quantities being given,

TAB. I. When the ſide of the polygon is 1.

Numb. of ſides.	Rad. of the circu. circ.	Rad. of the inſcri. circ.	The area.
3	0,5773503	0,2886751	0,4330127
4	0,7071068	0,5000000	1,0000000
5	0,8506508	0,6881910	1,7204774
6	1,0000000	0,8660254	2,5980762
7	1,1523825	1,0382617	3,6339124
8	1,3065630	1,2071068	4,8284271
9	1,4619022	1,3737387	6,1818242
10	1,6186340	1,5388418	7,6942088
11	1,7747329	1,7028437	9,3656404
12	1,9318516	1,8660254	11,1961524

TAB. II. When the radius of the circum. circle is 1.

Numb. of ſides.	Length of the ſide.	Rad. of the inſcri. circ.	The area.
3	1,7320508	0,5000000	1,2990381
4	1,4142136	0,7071068	2,0000000
5	1,1755705	0,8090170	2,3776412
6	1,0000000	0,8660254	2,5980762
7	0,8677674	0,9009689	2,7364102
8	0,7653668	0,9238795	2,8284271
9	0,6840403	0,9396926	2,8925437
10	0,6180340	0,9510565	2,9389263
11	0,5634651	0,9594931	2,9735250
12	0,5176381	0,9659259	3,0000000

Put N = tabular number of any column

not exceed 12; if there be conſidered,
3d. The radius of the inſcrib'd circle.
4th. The area of the polygon.
the other three are readily found by theſe Tables.

Tab. III. When the radius of the inſcrib. circle is 1.

Numb. of ſides.	Length of the ſide.	Rad. of the circum. cir.	The area.
3	3,4641016	2,0000000	5,1961524
4	2,0000000	1,4142236	4,0000000
5	1,4530851	1,2360680	3,6327128
6	1,1547005	1,1547005	3,4641016
7	0,9631491	1,1099160	3,3710222
8	0,8284271	1,0823919	3,3137084
9	0,7279405	1,0641776	3,2757315
10	0,6498394	1,0514622	3,2491970
11	0,5872521	1,0422172	3,2298913
12	0,5358984	1,0352760	3,2153904

Tab. IV. When the area is 1.

Numb. of ſides.	Length of the ſide.	Rad. of the circum. cir.	Rad. of the incrib. circ.
3	1,5196716	0,8773827	0,4386912
4	1,0000000	0,7071068	0,5000000
5	0,7623870	0,6485251	0,5246678
6	0,6204033	0,6204033	0,5372849
7	0,5245813	0,6045183	0,5446520
8	0,4550899	0,5946034	0,5493420
9	0,4021996	0,5879764	0,5525172
10	0,3605106	0,5833184	0,5547687
11	0,3267617	0,5799148	0,5564242
12	0,2988585	0,5773503	0,5576775

in either of theſe four tables.

PROPOSITION IX.

The ſide (S) *of a regular polygon being given,* *Fig.* 27. 28.

I. *To find the radius* (R) *of the circumſcribing circle.*

RULE. Multiply the given ſide by N, and the product is the radius ſought.

Ex. *What is the radius of a circle that can circumſcribe a regular octagon whoſe ſide is* 12?

Here N = 1,3065630, found in Tab. I.
Then 1,3065630 × 12 = 15,678756 = R.

II. *To find the radius* (r) *of the inſcrib'd circle?*

RULE. N, multiplied by the given ſide, gives r.

Ex. *If* S *is* 12, *what is* r?

Here N = 1,2071068, found in Tab. I.
Then 1,2071068 × 12 = 14,4852816 = r.

III. *To find* (A) *the area.*

RULE. N, multiplied by the ſquare of the given ſide, gives the area.

Ex. *If* S *is* 12, *what is* A?

Here N = 4,8284271, found in Tab. I.
Then 4,8284271 × 12 × 12 = 695,2935024 = A.

PROPOSITION X.

The radius (R) *of a circle circumſcribing a regular polygon being given.* Fig. 27. 28.

I. *To find* (S) *the ſide of that polygon.*

Rule. N, multiplied by the given radius, gives S.

Ex. *If the radius of a circle is* 12; *required the ſide of the inſcribed regular octagon?*

Here N = 0,7653668, found in Tab. II.
Then 0,7653668 × 12 = 9,1844016 = S.

II. *To find* (r) *the radius of a circle inſcrib'd in that polygon.*

Rule. N, multiplied by R gives r.

Ex. *If R is* 12; *what is r?*

Here N = 0,9238795, found in Tab. II.
Then 0,9238795 × 12 = 11,086554 = r.

III. *To find* (A) *the area of that polygon.*

RULE. N, multiplied by the ſquare of the given radius, gives the area ſought.

Ex. *If* $R = 12$; *what is A?*

Here N = 2,8284271, found in Tab. II.
Then 2,8284271 × 12 × 12 = 407.2935024 = A.

PROPOSITION XI.

The radius (r) *of a circle inſcrib'd in a regular polygon, being given.* Fig. 27. 28.

I. *To find* (S) *the length of the ſide of that polygon.*

RULE. N, multiplied by the given radius, gives S.

Ex. *What is the ſide of a regular octagon, circumſcribing a circle whoſe radius is* 12 *?*

Here N = 0,8284271, found in Tab. III.
Then 0,8284271 × 12 = 9,9411252 = S.

II. *To find* (R) *the radius of a circle that will circumſcribe the polygon.*

RULE. N, multiplied by r, gives R.

Ex. *If* r = 12; *what is* R?

Here N = 1,0823919, found in Tab. III.
Then 1,0823919 × 12 = 12,9887028 = R.

III. *To find* (A) *the area of that polygon.*

Rule. N, multiplied by the ſquare of r, gives A.

Ex. *If* r = 12; *what is* A?

Here N = 3,3137084, found in Tab. III.
Then 3,3137084×12×12=477,1740196=A.

PROPOSITION XII.

The area (A) *of a regular polygon being given.* Fig. 27. 28.

I. *To find* (S) *the length of its ſide.*

Rule. N, multiplied by the ſquare root of the given area, the product is the ſide ſought.

Ex. *What is the ſide of a regular octagon, whoſe area is* 144?

Here N = 0,4550899, found in Tab. IV.
Then 0,4550899×√144=5,4610588=S.

II. *To find* (R) *the radius of a circle circumſcribing that polygon.*

RULE. N, multiplied by the ſquare root of A, gives R.

EX. *If* $A = 144$; *what is* R?

Here $N = 0,5946034$, found in Tab. IV.
Then $0,5946034 \times \sqrt{144} = 7,1352408 = R$.

III. *To find* (r) *the radius of a circle inſcrib'd in that polygon?*

RULE. N, multiplied by the ſquare root of A gives r.

EX. *If* $A = 144$; *what is* r?

Here $N = 0,549342$, found in Tab. IV.
Then $0,549342 \times \sqrt{144} = 6,592104 = r$.

PROPOSITION XIII.

The diameter of a circle being known.

I. *To find the circumference.*

RULE. Multiply the diameter by 3,1416 ($=p$) and the product is the circumference.

Ex. *If the diameter is* 12; *what is the circumf.*

Then ($12\times3,1416=$)37,6992 is the circumf.

II. *To find the area.*

Rule. Multiply the ſquare of the diameter by 0,7854 ($\frac{1}{4}p$) and the product is the area.

Ex. *If the diameter is* 12, *what is the area?*

Then ($12\times12\times0,7854=$)113,0976 is the area.

Or. The radius, multiplied by half the circumference, gives the area of the circle.

Thus $\frac{37,6992}{2}\times6=113,0976$.

Again, the ſquare of the diameter multiplied by 0,392699 $\left(=\frac{1}{8}p\right)$ gives the area of a ſemicircle.

III. *To find the ſide of a ſquare equal in area to the circle.*

Rule. Multiply the diameter by 0,886217 ($=\frac{1}{2}\sqrt{p}$) and the product is the ſide of the ſquare.

Ex. *If the diameter is* 12; *what is the ſide of the equal ſquare?*

Then ($12\times0,8862=$)10,6344 is the ſide of the ſquare.

PROPOSITION XIV.

The circumference of a circle being known.

I. *To find the diameter.*

RULE. Multiply the circumference by 0,31831, $\left(=\frac{1}{p}\right)$ and the product is the diameter.

EX. *Suppoſe the circumference is* 12; *required the diameter?*

Then (12×0,31831=)3,81972 is the diameter.

II. *To find the area.*

RULE. Multiply the ſquare of the circumference by 0,0795776 $\left(=\frac{1}{4p}\right)$ and the product is the area.

EX. *Suppoſe the circum. is* 12; *required the area?*

Then (12×12×0,07958=)11,45952 is the area.

III. *To find the ſide of an equal ſquare.*

RULE. Multiply the circumference by 0,282095, $\left(=\frac{1}{2}\sqrt{\frac{1}{p}}\right)$ and the product is the ſide of the equal ſquare.

Ex. *Suppoſe the circumference is 12; required the ſide of a ſquare, of equal area to that circle?*

Then $(12\times0{,}2821=)3{,}3852$ is the ſide required.

PROPOSITION XV.

The area of a circle being known.

I. *To find the diameter.*

Rule. Multiply the ſquare root of the area by $1{,}12837$, $\left(=2\sqrt{\frac{1}{p}}\right)$ and the product will be the diameter.

Ex. *When the area is 12; what is the diameter?*

Then $(\sqrt{12}\times1{,}12837=)3{,}90877$ is the diameter.

II. *To find the circumference.*

Rule. Multiply the ſquare root of the area by $3{,}5449$, $(=2\sqrt{p})$ and the product will be the circumference.

Ex. *When the area is 12; what is the circumf.*

Then $(\sqrt{12}\times3{,}5449=)12{,}3798$ is the circumf.

III. *To find the ſide of a ſquare of equal area.*

Rule. The ſquare root of the given area, will be the ſide of the ſquare required.

Ex. *When the area is* 12; *what is the ſide of the equal ſquare.*

Then ($\sqrt{12}=$)3,4641 is the ſide required.

PROPOSITION XVI.

The ſide of a ſquare, or its area, being known.

I. *To find the diameter of a circle of equal area.*

Rule. Multiply the ſide of the ſquare, by 1,12837 $\left(=2\sqrt{\frac{1}{p}}\right)$ and the product will be the diameter ſought.

Ex. *What is the diameter of that circle, equal in area to a ſquare whoſe ſide is* 12?

Then ($12\times1,12837=$)13,54044 is the diameter required.

II. *To find the circumference of an equal circle.*

Rule. Multiply the ſide of the ſquare by 3,5449, ($=2\sqrt{p}$) and the product will be the circumference ſought.

Ex. *What is the circumference of that circle, whoſe area is equal to a ſquare wherein the ſide is* 12?

Then ($12\times3,5449=$)42,5388 is the circum. requ.

III. *To find the ſide of a ſquare, that may be inſcrib'd in that circle of equal area to the given ſquare.*

RULE. Multiply the given ſide by 0,797884, $\left(=\sqrt{\frac{2}{p}}\right)$ and the product is the ſide of the ſquare ſought.

EX. *What is the ſide of that ſquare, which may be in ſcribed in a circle of equal area to a ſquare whoſe ſide is* 12?

Then (12×0,797884=)9,574608 is the ſide requ.

IV. *To find the area of a ſquare, that may be inſcrib'd in a circle of equal area to the given ſquare.*

RULE. Multiply the ſquare of the given ſide by 0,63662, $\left(=\frac{2}{p}\right)$ and the product is the area of of the ſquare required.

EX. *What is the area of that ſquare, which may be inſcrib'd in a circle of equal area to a ſquare whoſe ſide is* 12?

Then (12×12×0,63662=)91,67328 is the area required.

PROPOSITON XVII.

The radius (CA) *of a circle, and the chord* (AB) *of an arc thereof being known: To find the verſed ſine* (DE) *of half that arc.* Fig. 31.

RULE. From the ſquare of the radius, take the ſquare of half the chord; the ſquare root of the remainder, ſubtracted from the radius, leaves the verſed ſine.

Or $DE = CA - \sqrt{\overline{CA}^2 - \overline{AE}^2}$.

Ex. *In a circle whoſe radius is* 25: *What is the verſed ſine of that arc, the chord of whoſe double is* 48?

Now $25 \times 25 - 24 \times 24 = 49$; whoſe ſquare root is 7.

Then $(25 - 7 = 18)$ is the verſed ſine required.

PROPOSITION XVIII.

The radius (CA) *of a circle, and the verſed ſine* (ED) *of an arc* (BD) *thereof being known.* Fig. 33.

I. *To find the chord* (AB) *of twice that arc.*

RULE. From twice the radius, take the verſed ſine; multiply the remainder by the verſed ſine; then will twice the ſquare root of the product give the chord of twice the arc.

Or $BA = 2\sqrt{\overline{DF - DE} \times DE}$

Ex. *In a circle whoſe radius is* 25; *what is the chord of twice the arc, whoſe verſed ſine is* 18?

Then $(\sqrt{\overline{2 \times 25 - 18} \times 18} \times 2 =)$ 48 the chord required.

II. *To find the chord* (BD) *of that arc.*

Rule. Multiply twice the radius by the verſed ſine, and the ſquare root of the product will be the chord of the arc.

Or BD $= \sqrt{DF \times DE}$.

Ex. *If the radius of a circle is 25, and the verſed ſine of an arc thereof, is 18: What is the chord of that arc?*

Then ($\sqrt{2 \times 25 \times 18} =$) 30 is the chord required.

PROPOSITION XIX.

The chord (AB) *of any circular arc, and the verſed ſine* (ED) *of half that arc being known.*

I. *To find the radius* (CA) *of the circle.* Fig. 33.

Rule. To the ſquare of half the chord, add the ſquare of the verſed ſine; divide the ſum by twice the verſed ſine, and the quotient is the radius required.

Or CA $= \overline{\overline{BE}^2 + \overline{DE}^2} \div 2\,DE$.

Ex. *If the chord* AB $= 48$, *and the verſed ſine* DE $= 18$; *what is the radius of the circle?*

Then $\left(\dfrac{\frac{48}{2} \times \frac{48}{2} + 18 \times 18}{2 \times 18} =\right)$ 25 is the radius required.

Note, The diſtance of the chord from the centre, is found by ſubtracting the verſed ſine from the radius.

II. *To find the chord* (BD) *of half the arc.*

Rule. To the ſquare of half the chord, add the ſquare of the verſed ſine; and the ſquare root of the ſum, will be the chord of half the arc.

Or $BD = \sqrt{\overline{BE}^2 + \overline{DE}^2}$.

Ex. *If the chord is* 48, *and the verſed ſine is* 18; *what is the chord of half the arc?*

Then ($\sqrt{24 \times 24 + 18 \times 18} =$) 30 is the chord of half the arc.

PROPOSITION XX.

To find the length of a circular arc (BDA) Fig. 33.

I. *When the chord* (AB) *of that arc, and the chord* (BD) *of its half are known.*

Rule. From 8 times the chord of half the arc, ſubtract the chord of the whole arc; and $\frac{1}{3}$ of the remainer, will be the length of the arc nearly.

Arc $ADB = \frac{1}{3} \cdot \times \overline{8\,BD - AB}$.

Ex. *If the chord of the arc is* 48, *and the chord of half the arc is* 30; *what is the length of the arc?*

Then $\left(\frac{30\times8-48}{3}=\right)$ 64 is the length of the arc.

II. *When the chord* (AB) *of an arc,* (ADB) *and the verſed ſine* (DE) *of half the arc are known.*

Rule. Find the diameter (by Caſe I. Prop. XIX.) divide $\frac{2}{3}$ of the verſed ſine, by the diameter leſſened by $\frac{82}{100}$ of the verſed ſine; the quotient added to 1, and the ſum multiplied by the chord, will give the length of the arc very near.

$$\text{Arc ADB} = \overline{\frac{\frac{2}{3}DE}{DF-\frac{82}{100}DE}+1}\times AB.$$

Ex. *If the chord of an arc is* 48, *and the verſed ſine of half the arc is* 18: *What is the length of that arc.*

Now $\left(\frac{24\times24+18\times18}{18}=\right)$ 50 is the diameter.

And $(50-\frac{82}{100}\times18=)$35,24 is the diviſor.

Then $\left(1+\frac{\frac{2}{3}\times18}{35,24}\times48=\right)$ 64,34496 is the length of the arc.

III. *When the diameter* (DF) *of a circle, and the arc* (ADB) *in degrees are known.*

Rule. Multiply the degrees in the arc, by the diameter of the circle; the product multiplied by 0,0087267 $\left(=\frac{p}{360}\right)$ will ſhew the length of the arc in that kind of meaſure the diameter is of.

Ex. *In a circle whoſe diameter is* 50 *feet; what is the length of an arc of* 147 *degrees,* 29 *minutes?*

Now 147 deg. 29 min. = 147,483 degrees.
Then $(147,483\times50\times0,0087267=(64,352$ feet, is the length of the arc required.

Note, The arc in degrees may be eaſily found, by having the diameter of the circle, and the length of the arc, found by either of the foregoing Rules. Thus.

Divide the number 114,59132 $\left(=\frac{360}{p}\right)$ by the diameter, the quotient multiplied by the length of the arc, gives the arc in degrees.

PROPOSITION XXI.

To find the area (CADB) *of a circular ſector.* Fig. 34. 35.

I. *When the radius* (CA) *of the circle, and the length of the ſectoral arc* (ADB) *are known.*

Rule. Multiply the radius by half the given arc, and the product will be the area of the ſector.

Ex. *In a circle whoſe radius is* 25; *required the area of a ſector on the arc whoſe length is* 64,352?

Then $\left(\frac{64,352}{2}\times25=\right)$804,4 is the area required.

II. *When the radius of the circle, and the degrees in the ſectoral arc known.*

Rule. Multiply the given degrees, by the ſquare of the radius; the product multiplied by 0,0087267, will give the area of the ſector.

Ex. *In a circle whoſe radius is* 25 *feet: What is the area of a ſector on an arc of* 147 *degrees,* 29 *minutes?*

Now 147 deg. 29 min. = 147,483.
Th. (147,483×25×25×0,0087267=)804,4117 is the area of the ſector required.

PROPOSITION XXII.

To find the area (ADBA=A) *of a circular ſegment, whoſe height,* (DE=v) *and chord* (AB=b) *of its arc are known.* Fig. 36.

Rule. I. Multiply the height by 0,626; to the ſquare of the product, add the ſquare of half the chord: Multiply twice the ſquare root of the ſum, by two thirds of the height, and the product is the area.

$$\text{Or } A = \tfrac{2}{3}v \times 2\sqrt{\overline{0,626\,v}|^{2} \times \overline{\tfrac{1}{2}b}|^{2}}$$

Ex. *What is the area of that circular ſegment whoſe height is* 18; *and the chord of whoſe arc is* 48?

Now 18 × 0,626 = 11,268.
And $11,268 \times 11,268 + \frac{48}{2} \times \frac{48}{2} = 702,967824$ whoſe ſquare root is 26,5135.

Then $\left(26,5135 \times 2 \times \frac{2\times 18}{3} =\right)$ 636,324 is the area of the ſegment.

Rule. II. *Find the chord* (AD=h) *of half the arc.* (by Caſe II. Prop. XIX.) Then,

To the ſquare of the chord, add the ſquare of the height; to twice the ſquare root of the ſum, add the chord of half the arc; multiply the ſum by $\frac{4}{15}$ of the height, and the product will give the area.

Or $A = \overline{2\sqrt{bb+vv}+h}\times\frac{4}{15}v$.

Ex. *In a circular ſegment whoſe height is* 18, *and the chord of the arc is* 48: *What is the area?*

Now ($\sqrt{24\times24+18\times18}=$) 30, is the chord (AD) of half the arc.

And $\sqrt{48\times48+18\times18}=51,26402$.

Then ($\overline{51,26402\times2+30}\times18\times\frac{4}{15}=$) 636,13459 is the area of the ſegment.

Rule. III. Find the diameter (by Caſe I. Prop. XIX.) divide 32 times the height, by 80 times the diameter leſſened by 15 times the height; take the quotient from the number $\frac{4}{3}$; multiply the remainder by the ſquare root of the product of the diameter and height; this product multiplied by the height, will give the area of the ſegment.

Or $A = v\sqrt{dv}\times\overline{\frac{4}{3}-\frac{32v}{80d-15v}}$

Ex. *What is the area of a circular ſegment, the height being* 18; *and the chord of the arc* 48?

Now $\left(\frac{24\times24+18\times18}{18}\right)$ 50 is the diameter.

And $18\times32=576$ is the dividend.
And $50\times80-18\times15=3730$ is the diviſor.

Then $\frac{576}{3730}=0,15469$, *&c.*

Therefore ($\overline{\tfrac{4}{3}-0,15469}\times\sqrt{\overline{50\times18}}\times18=$) 636,444 is the area of the ſegment.

Rule. IV. Find the radius (r) (by Caſe I. Prop. XIX.) and the arc (a) in degrees (by the note to Prop. XX.) multiply the ſquare of the radius, the arc in degrees, and the number 0,0087267 continually; call the product A.

From the ſquare of the radius, take the ſquare of half the chord; multiply the ſquare root of the remainder by half the chord; call the product B.

Then B taken from A, will leave the area of the Segment.

$$\text{Or } rr\times a\times\frac{3,1416}{360}-\tfrac{1}{2}b\sqrt{rr-hh}=\text{A}.$$

Ex. *Suppoſe the chord of the arc of a circular ſegment is* 48, *and the height* 18; *what is its area?*

Now $\left(\frac{24\times24+18\times18}{2\times18}=\right)$ 25 is the radius.
And 147,483 are the degrees in the arc.
Then ($25\times25\times147,483\times0,0087267=$) $804,3937=\text{A}$.
And ($\tfrac{48}{2}\times\sqrt{25\times25-\tfrac{48}{2}\times\tfrac{48}{2}}=$) $168=\text{B}$.
Therefore ($\text{A}-\text{B}=$) 636,3937 is the area of the ſegment required.

PRO-

PROPOSITION XXIII.

In a circular zone (BDdb) or that part of a circle contain'd between two parallel chords, (BD=2B,bd=2b) the length of thoſe chords, and their diſtance (Aa = h) being known. **Fig. 32.**

I. *To find the diſtance* (CA=*x*) *of the centre* (C) *of that circle, from the middle* (A) *of the greater chord.*

RULE. To the ſquare of the diſtance of the chords, add the ſquare of half the leſſer chord.

The difference between this ſum and the ſquare of half the greater chord, divided by twice the diſtance of the chords, will give the diſtance of the centre as required.

$$\text{Or } x = \frac{\pm BB \mp bb + hh}{2b}$$

Note, If the ſaid ſum

$$\text{is} \left\{ \begin{array}{l} \text{greater} \\ \text{leſs} \\ \text{equal to} \end{array} \right. \left.\begin{array}{l} \\ \\ \end{array}\right\} \text{than} \left\{ \begin{array}{l} \\ \\ \\ \end{array} \right. \text{the ſquare of the half chord,}$$

$$\text{the centre falls} \left\{ \begin{array}{l} \left.\begin{array}{l} \text{between} \\ \text{without} \end{array}\right\} \text{the two chords.} \\ \text{in the middle of the greater chord.} \end{array} \right.$$

EX. *Suppoſe the greater chord is* 48, *the leſſer* 30; *and their diſtance* 13: *How far is the centre of that circle diſtant from the middle of the greater chord.*

Now $13 \times 13 + 15 \times 15 = 394$.
And $24 \times 24 = 576$.

Then $\left(\frac{576 - 394}{13 \times 2} =\right)$ 7 the diſtance of the centre as required.

Here the two chords are on the ſame ſide from the centre.

Note, The diſtance of the greater chord from the centre of the circle being known; the radius of that circle may be thus found.

RULE. To the ſquare of half the greater chord, add its diſtance from the centre; the ſquare of the ſum, will be the radius required.

EX. *Thus, in the foregoing example, where half the greater chord is* 24; *and its diſtance from the centre is found to be* 7.

Then $(\sqrt{24 \times 24 + 7 \times 7} =)$ 25 is the radius of the circle.

II. *To find the height* ($aE = v$) *of the circular ſegment* (bEd) *whoſe baſe* (bd) *is the leſſer chord.*

RULE. To the ſquare of half the greater chord, add the ſquare of the breadth of the zone; from the ſum take the ſquare of half the leſſer chord; divide the remainder by the breadth; call the quotient, A.

To the ſquare of half the leſſer chord; add the ſquare of $\frac{1}{2}$ A; from the ſquare root of the ſum, take $\frac{1}{2}$ A; and the remainder is the verſed ſine, or height of the ſegment required.

$$\text{Or } A = \frac{BB + hh - bb.}{h}$$

$$\text{Then } v = \sqrt{bb + \overline{\tfrac{1}{2} A}|^{2}} - \tfrac{1}{2} A.$$

Ex. *Suppoſe the greater chord is* 48, *the leſſer* 30; *and their diſtance is* 13: *What is the height of the ſegment whoſe baſe is the leſſer chord.*

$$\text{Now } \left(\frac{\overline{24 \times 24 + 13 \times 13 - 15 \times 15}}{14} = \right) 40 = A;$$

its half = 20.

And 15×15+20×20=625; whoſe ſquare root, is 25.

Then (25 — 20 =) 5 is the height of the ſegment ſtanding on the leſſer chord.

And (13 + 5 =) 18 is the height of the ſegment whoſe baſe is the greater chord.

Note, The height of either of the ſegments, ſtanding on the greater or leſſer chords being known, the radius of the circle may be found by Caſe I. Prop. XIX.

PROPOSITION XXIV.

To find the area of a circular zone; its breadth and the length of its ends being known.

RULE. I. Find the heights of (AE,*a*E) the circular ſegments (BED,bEd) on each end (BD,bd) by Caſe II. Prop. XXIII.

Find the diameter (by Caſe I. Prop. XIX.)

Then the difference of the ſegments on the greater and leſſer ends of the zone (found by Rule III. Prop. XXII) will be the area of the zone.

Ex. *What is the area of a circular zone; one end being* 48, *the other end* 30; *and the breadth* 13?

By Caſe II. Prop. XXIII. the height of the ſegment on the leſſer end of the zone is equal to 5.

And $13+5=18$ is the height of the ſegment on the greater end.

By Caſe I. Prop. XIX. the diameter is equal to 50.

Then 636,444 is the area of the ſegment, on the greater end of the zone by Rule III. Prop. XXII.

And 102,1865 is the area of the ſegment, on the leſſer end, by the ſame.

Conſequently 534,257 is the area of the zone.

RULE. II. 1. Find the diſtance of the centre of the circle from the greater end of the zone (by Caſe I. Prop. XXIII.)

2. Find the radius, (by the note to the ſame.)

3. Find the chord of the arc, between the ends of the zone; (Caſe I. Prop. IV.) and the verſed ſine of half the arc. (by Prop. XVII.)

4. Find the length of the arc (by Caſe II. Prop. XX.)

5. Multiply half the leſſer end of the zone, by the breadth thereof, call the product A.

Multiply the radius by the length of the arc, call the product B.

7. From

7. $\left.\begin{matrix}\text{From}\\ \text{To}\end{matrix}\right\}$ the ſum of A and B $\left\{\begin{matrix}\text{take}\\ \text{add}\end{matrix}\right\}$ the product of the diſtance from the centre, by the $\left\{\begin{matrix}\text{diff.}\\ \text{ſum}\end{matrix}\right\}$ of half the ends of the zone; and the $\left\{\begin{matrix}\text{remainder}\\ \text{ſum}\end{matrix}\right\}$ will be the area of the zone; when the centre falls $\left\{\begin{matrix}\text{without}\\ \text{between}\end{matrix}\right\}$ the ends.

E x. *In a circular zone, ſuppoſe the greater end is* 48, *the leſſer end* 30, *and the breadth* 13: *What is area?*

Now $\left(\frac{24\times24-\overline{15\times15+13\times13}}{2\times13}=\right)7$ is the diſtance of the greater chord from the center (which falls without the zone.)

And $(\sqrt{24\times24+7\times7}=)$ 25 is the radius.

Alſo $24-15=9$ is the difference between the half ends.

Then $(\sqrt{9\times9+13\times13}=)15,8114$ is the chord of the arc between the ends of the zone.

And $\left(25-\sqrt{25\times25-\frac{15,8114}{2}\times\frac{15,8114}{2}}=\right)$ 1,283 is the verſed ſine of half that arc.

Then $\left(1+\frac{\frac{2}{3}\times1,283}{50-0,82\times1,283}\times15,8114=\right)16,0877$ is the length of the arc between the two ends of the zone.

Now $(\frac{30}{2}\times13=)195=A$.

And $(16,0877\times25=)402,182=B$.

And $(402,182+195=(597,182$ is the ſum of A and B.

Then $(597,182-9\times7=)534,182$ is the area of the zone.

Rule. III Find the area of one of the two circular ſegments contain'd in the given zone. (by Caſe II. Prop. XXII.)

Multiply half the ſum of the ends of the zone, by the breadth; to the product add twice the area of the ſegment, before found; and the ſum will be the area of the zone.

Ex. *In a circular zone*; *ſuppoſe the greater end is* 48, *the leſſer end* 30; *their diſtance*, 13; *the baſe of one of the contain'd circular ſegments is* 15,8114, *and its height* 1,283: *What is the area of that zone?*

Now $1,283 \times 0,626 = 0,803158$.

And $0,803158 \times 0,803158 + \frac{15,8114}{2} \times \frac{15,8114}{2}$ $= 63,145062$; whoſe ſquare root is 7,9464.

And $\left(7,9464 \times 2 \times \frac{2 \times 1,283}{3} = \right) 13,5936$ the area of one ſegment.

Then $\left(\frac{48+30}{2} \times 13 + 13,5936 \times 2 = \right) 534,1872$ is the area of the zone.

PROPOSITION XXV.

To find the diameter of a circle, whoſe area ſhall be in a given proportion to that of a circle whoſe diameter is known.

RULE.

The given diameter $\left\{\begin{matrix}\text{multiply'd}\\ \text{divided}\end{matrix}\right\}$ by the ſquare root of the intended $\left\{\begin{matrix}\text{increaſe,}\\ \text{decreaſe,}\end{matrix}\right\}$ will give the diameter of the circle required.

Ex. I. *What is the diameter of a circle, whoſe area is* 9 *times as much as one of* 21 *inches diameter?*

Then $(21 \times \sqrt{9} =)$ 63 inches is the diameter of a circle 9 times as large as one 21 inches in diameter.

Ex. II. *What is the diameter of a circle, whoſe area is but* $\frac{1}{9}$ *of a circle of* 21 *inches diameter?*

Then $\left(\frac{21}{\sqrt{9}} =\right)$ 7 inches, is the diameter of the circle required.

PROPOSITION XXVI.

To find the area of figures, whoſe ſides are partly right lines, and partly arcs of a circle.

RULE.

To every curved ſide draw a chord; and the given figure will be reduced to a right lined one, and as many ſegments as there were curved ſides. Then the ſum of the areas of theſe parts, will be the ſuperficial content of the given figure.

EXAMPLE.

Suppoſe a field of five ſides, two whereof are the arches of circles; now the chords being drawn, the figure will be a five-ſided right-lined figure, which being (by lines drawn) divided into a trapezium and a triangle; in the trapezium, the diagonal is 7 *chains, and the ſum of the perpendiculars* $6\frac{1}{2}$ *chains; in the triangle, the baſe is* 4 *chains, and the perpendicular* 6 *chains; in one ſegment, the whole chord is* 6 *chains, and the chord of half the arch is* $3\frac{1}{2}$ *chains; in the other ſegment the whole chord is* 4 *chains, and the chord of half the arch is* $2\frac{1}{2}$ *chains; what is the area of this figure.* Fig. 37.

Firſt, $\frac{7}{2}=3,5$, then $3,5\times6,5=22,75$, the area of the trapezium;

Secondly, $\frac{6}{2}=3$, then $3\times4=12$, the area of the triangle;

Thirdly, In one ſegment the chord of half the arch is 3,5, and half the whole chord is 3, then $3,5\times3,5=12,25$; and $3\times3=9$; and $12,25-9=3,25$; whoſe ſquare root is 1,8 the verſed ſine.

And $\frac{6}{2}\times\frac{6}{2}+\frac{1,8\times1,8}{4}=9,81$; whoſe ſquare root is 3,132.

Then $\frac{3,132\times8+3,5\times2}{15}\times1,8\times2=7,6932$ the area ℘ Rule II. Prop. XXII.

Fourthly, In the ſecond ſegment, the chord of half the arc is 2,5; and half the whole chord is 2.

Then $\sqrt{2,5\times2,5-2\times2}=1,5$ the verſed ſine.

And $\frac{4}{2}\times\frac{4}{2}+\frac{1,5\times1,5}{4}=4,5625$; whoſe ſquare root is 2,136.

Then $\frac{2,136\times8+2,5\times2}{15}\times1,5\times2=4,416$ the area (by Rule II. Prop. XXII.

Now $21,75+12+7,693+4,416=45,859$ ſquare chains for the area of the field.

And as 10 ſquare chains make 1 acre.

Therefore $\frac{45,859}{10}=$ 4 acres 2 roods 14 poles.

SECTION VII.

PRACTICAL QUESTIONS.

QUESTION I.

A Round pillar 7 inches over, is ſufficient to carry a certain weight; of what diameter is the column that contains 10 times the ſtone on the ſame length?

Now $7\times7\times10=490$, whoſe ſquare root is 22,135 inches, the diameter required.

QUESTION II.

A brewer has a ciſtern which is fill'd by three pipes, each of 3 inches bore; of what diameter muſt the bore of that pipe be, which in the ſame time, will throw him in $2\frac{1}{2}$ times as much water?

The quantity of water thrown in, being as the ſquares of the diameters;

Therefore $3\times3\times3\times2,5=67,5$, whoſe ſquare root is 8,215 inches, *&c.* the diameter ſought.

QUESTION III.

If a peice of a cable, 3 feet long and 9 inches in compaſs weighs 22 ℔; *what will a fathom weigh, of that cable whoſe diameter is 9 inches?*

Now 1 *circumf.* : 0,31831 *diam.* : : 9 *circumf.* : 2,86479 *diam.* whoſe ſquare is 8,2069, *&c.*

Then 8,2069 : 22 ℔ : : 81 (9 × 9) : 217,14, which × by 2, gives 434,28, the weight ſought.

QUESTION IV.

I want in a garden a circular pond, that ſhall juſt take up half an acre; how long muſt the cord be that will ſtrike the circle?

The half acre contains 2420 ſquare yards;

Therefore 1 : 1,2723 : : 2420 : 3081,1441 the ſquare of the diameter, whoſe ſquare root is 55,508 the diameter; the half of which, 27,75 yards, is the length of the line ſought.

QUESTION V.

A carpenter is to put an oaken curb to a round well, at 8 d. ꝑ *foot ſquare; the breadth of the curb is to be* $7\frac{1}{4}$ *inches, and the diameter within is* $3\frac{1}{2}$ *feet; what will be the expence?*

Now $3\frac{1}{2}$ *f.* = 42 *in.*

Then 42 × 42 × 0,7854 = 1384,4456, the area within the curb.

Alſo 42 + 7,25 + 7,25 = 56,5, the outſide diameter of the curb.

And $56,5 \times 56,5 \times ,7854 = 2507,19315$ the area of a circle including the well and curb.

Then $2507,19315 - 1385,4456 = 1121,74755$, the area of the curb.

Now 8 *d.* = 0,03 £.

Ther. as $144 : 0,03 :: 1121,74755 : 0,259671$ £. = 5 *s.* 2¼ *d.* the expence ſought.

Or. The diameters of two circles being known, the difference of their areas may be found by the following.

RULE. Multiply the ſum of the diameters, the difference of the diameters, and 0,7854 continually; the product will be the difference of the areas.

Thus $\overline{56,5 + 42} \times \overline{56,5 - 42} \times 0,7854 = 1121,74755$ is the area of the curb.

QUESTION VI.

There is wanted in a garden a circular pond with a circular iſland in the middle; the diameter of the pond muſt be 100 *yards, and the circumference of the iſland the ſame; what will the digging of the pond come to at* 18 *d.* ℘ *foot ſquare on the ſurface?*

Now $100 \times 100 \times 0,7854 = 7854$ yards, the area of the pond and iſland.

And $100 \times 100 \times 0,07958 = 795,8$, the area of the iſland.

Then $7854 - 795,8 = 7058,2$ yards, the area of the pond = 63523,8 feet.

And $1 : ,075$ £. $:: 63523,8 : 4764,285$ = 4764 £. 5 *s.* 8½ *d.*

QUESTION VII.

Suppoſe ehe expence of paving a ſemicircular plot, at 2 *s.* 4 *d.* ꝑ *foot, amounted to* 10 *£. what is the diameter thereof?*

Now 2 *s.* 4 *d.* = 0,116 *£.*

Thereforeo, 116 *£.* : 1 *f.* : : 10 *£.* : 85,7142 the ſemicircle's area.

And 85,7142 × 2 = 171,4284, the circle's area.

Then 1 : 1,2732 : : 171,4284 : 218,26285713, whoſe ſquare root is 14,7734 the diameter ſought.

QUESTION VIII.

Suppoſe St. James's *ſquare to be* 180 *yards long, and* 150 *yards broad, in which there is a regular octagonal gravel walk, one of whoſe ſides is (ſuppoſe)* 28 *yards; what did the paving the reſt with purbeck ſtone come to, at* 3 *s.* 6 *d.* ꝑ *yard?*

Now 180 × 150 = 27000 yards, the area of the ſquare.

And 28 × 28 × 4,828427 = 3785,486768 yards the area of the octagon.

Then 27000 yards − 3785,486768 = 23214,513232 yards, what was paved.

And 1 *yard* : 0,175 *£.* : : 23214,513232 yards : 4062,5397; = £. 4062. 10 *s.* 9½ *d.*

QUESTION IX.

What is the area of the ſegment of a circle whoſe diameter is 50 *inches; ſuppoſing the ſection made* 14 *inches from the centre?*

Now $\frac{50}{2} = 25$, the radius, and $25 - 14 = 11$, the verſed ſine or height.

Then by Prop. XXII.

$$\frac{11 \times 32}{80 \times 50 - 15 \times 11} = 0{,}072802.$$

And $\frac{4}{3} - 0{,}072802 = 1{,}260531$.

And $\sqrt{50 \times 11} = 23{,}452$.

Then $(1{,}260531 \times 23{,}452 \times 11 =)\ 325{,}18156$ is the area of the ſegment.

QUESTION X.

A, B and C, bought a circular cheeſe, 14 *inches in diameter, which coſt them* 7 *s.* 6 *d. whereof A pays* 1 *s.* 4 *d. B* 2 *s.* 10 *d. and C* 3 *s.* 4 *d. now they agree that it ſhall be divided from the centre to the circumference; that is, it ſhould be cut into three ſectors; whoſe areas ſhould bear the ſame proportion to each other, as the prices paid; what part of the circumference will fall to each man's ſhare, together with the areas.*

Now $14 \times 14 = 196$, and $1 : {,}7854 :: 196 : 153{,}9384$, the whole area.

Alſo 7 *s.* 6 *d.* $= {,}375$ £. 1 *s.* 4 *d.* $= {,}0\cancel{6}$ £. 2 *s.* 10 *d.* $= {,}141\cancel{6}$ £. 3 *s.* 4 *d.* $= {,}1\cancel{6}$ £.

And $\frac{153{,}9384}{0{,}375} = 410{,}5$.

$$\text{Then}\ \left.\begin{aligned} 410{,}5 \times 0{,}0\cancel{6} &= 27{,}3\cancel{6} &&= A\text{'s} \\ 410{,}5 \times 0{,}141\cancel{6} &= 58{,}1541\cancel{6} &&= B\text{'s} \\ 410{,}5 \times 0{,}1\cancel{6} &= 68{,}41\cancel{6} &&= C\text{'s} \end{aligned}\right\}\ \text{ſhare of the area.}$$

Again, $1 : 1416 :: 14 : 43{,}9824 =$ circumf.

And

$$\text{And } \frac{43,9824}{0,375} = 117,286.$$

$$\left.\begin{array}{l} \text{Then } 117,286 \times 0,0\cancel{6} = 7,919 = A\text{,s} \\ 117,286 \times 0,141\cancel{6} = 16,615 = B\text{'s} \\ 117,286 \times 0,1\cancel{6} = 19,547 = C\text{'s} \end{array}\right\} \begin{array}{l} \text{ſhare} \\ \text{of the} \\ \text{circu.} \end{array}$$

QUESTION XI.

A, B and C, bought a grinding ſtone of 21 *inches in diameter; each paying a third; what part of the diameter muſt each grind down?*

This queſtion is anſwer'd, by reckoning each man to grind away one third of the circular area.

Now 21 × 21 × ,7854 = 346,3614, the area.

Then $\frac{346,3614}{3}$ = 115,4538, each man's area; which taken from the whole, leaves 230,9076, for the area of the circle remaining, when one man has ground away his ſhare.

And 1 : 1,27324 : : 230,9076 : 294, whoſe ſquare root is 17,14, the diameter of two men's ſhares next the centre.

Again, 1 : 1,2732 : : 115,4538 : 147, whoſe ſquare root is 12,14, the diameter of one man's ſhare next the centre.

Then 21 − 17,14 = 3,86 is the breadth of the ring for the 1ſt man's ſhare.

Alſo 17,14 − 12,14 = 5 is the breadth of the ring which the 2d man is to grind away.

And 12,14 is the diameter of the 3d man's ſhare.

QUESTION XII.

A workman is employ'd to ſet up a rail round a circular baſon at 12 *feet diſtance; and to lay a gravel walk between the rail and baſon: The price of the rail at* 5 *s. a linear yard, and the walk at* 1 *s.* 6 *d. a ſquare yard: Now a line of* 28,1267 *yards ſtretch'd cloſe by the brink of the baſon, will with both ends touch the rail: What will the workman's bill amount to?*

Now $\dfrac{\overline{28{,}1267}|^2}{2\ |} = 197{,}\cancel{7}$.

Then $\dfrac{197{,}\cancel{7}}{4} + 4 = 53{,}\cancel{4}$ the diameter of the rail.

And $1 : 3{,}1416 : : 53{,}\cancel{4} : 167{,}9010\cancel{6}$ or $167{,}9$, the circumference or length of the rail.

Therefore $\dfrac{53{,}\cancel{4}}{2} \times \dfrac{167{,}9}{2}, = 2243{,}330\cancel{5}$ the area including the walk and baſon.

Now 12 f. = 4 yards.

And $53{,}\cancel{4} - 4 \times 2 = 45{,}\cancel{4}$ the diameter of the baſon.

Then $\overline{53{,}\cancel{4} + 45{,}4} \times \overline{53{,}\cancel{4} - 45{,}\cancel{4}} \times 0{,}7854 = 621{,}33342$ ſquare yards is the area of the gravel walk.

Now $167{,}9 \times 0{,}25$ £. $= 41{,}975$ £. the price of the rail.

And $621{,}33342 \times 0{,}075$ £. $= 46{,}6$ the price of the walk.

Therefore 88 £. 11 *s.* 6 *d.* is the whole expence.

QUESTION XIII.

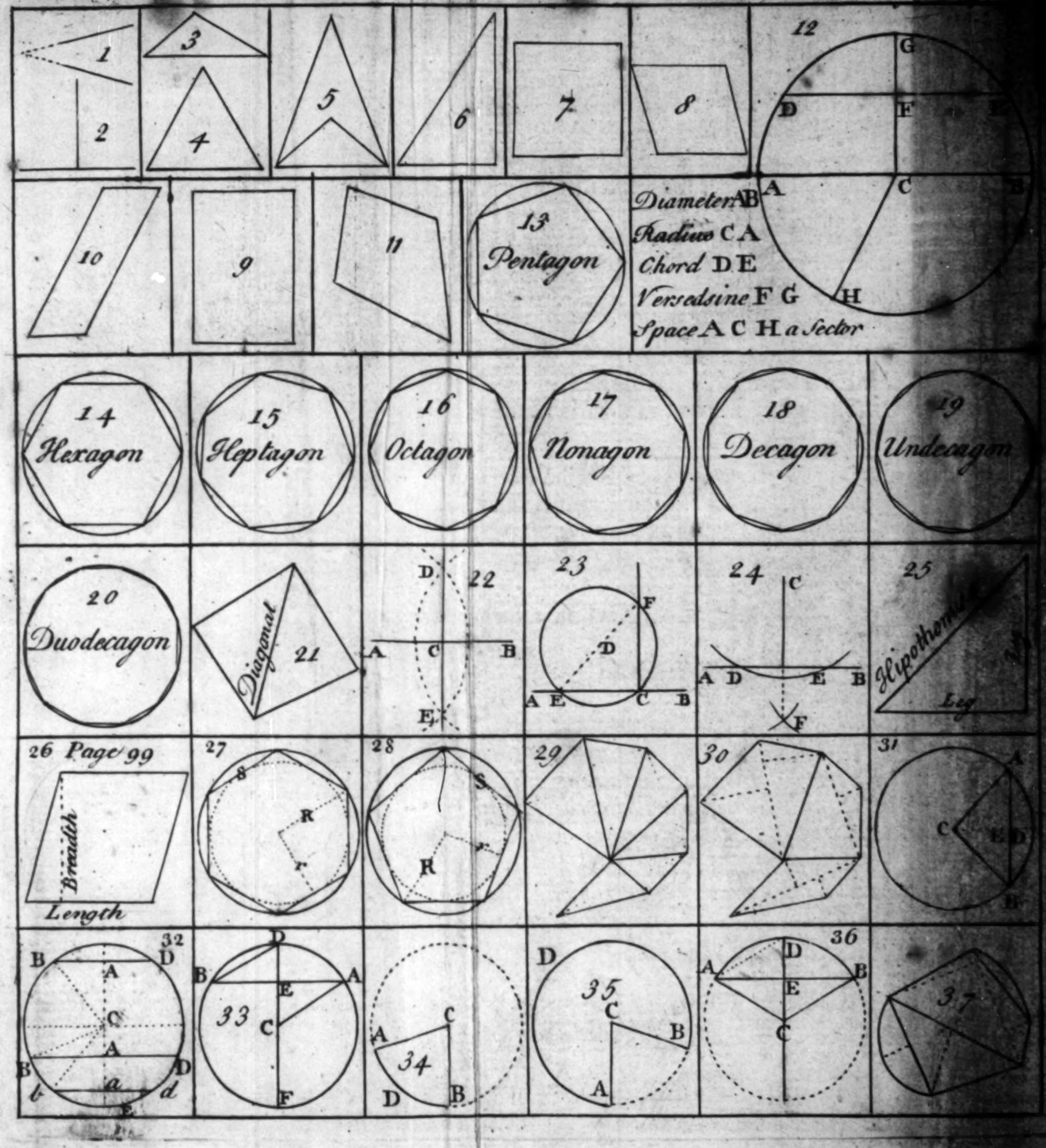
A gentleman has ordered a ſquare plat of two acres to be laid out fronting his houſe; in this plat he would have a regular octagonal baſon, containing a quarter of an acre, and four of its ſides parallel to the ſides of the plat; alſo gravel walks of 8 yards wide running diagonally, and from the middle of each ſide, terminating in a walk of the ſame breadth, ſurrounding the baſon: What will the expence of gravelling theſe walks come to, at 16 d. a ſquare yard?

Now 4840 yards = 1 acre.

Therefore $\sqrt{4840 \times 2} = 98{,}3869886$ is the ſide of the plat.

Alſo

Plate. I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Pentagon
Diameter AB
Radius CA
Chord DE
Versedsine FG
Space ACH a Sector
14
Hexagon
15
Heptagon
16
Octagon
17
Nonagon
18
Decagon
19
Undecagon
20
Duodecagon
Diagonal
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B
A
36
A
D
B
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C
37

Alſo $\left(\frac{\sqrt{4840}}{4}=\right) 34{,}7850543 \times 0{,}4550890 = 15{,}8303257$, is the ſide of the octagon.

And $\frac{\sqrt{4840}}{4} \times 0{,}549342 = 19{,}1088902$ is the radius of the inſcrib'd circle.

Therefore $\frac{98{,}3869886}{2} - 19{,}1088902 - 8 = 22{,}0846041$ is the length of one of the walks from the ſides of the plat.

Then $22{,}0846071 \times 8 \times 4 = 706{,}7074592$ is the area of thoſe four walks.

Again, $\sqrt{4840 \times 2 \times 2} = 139{,}140217$ is the diagonal of the ſquare plat.

And $\frac{139{,}140217}{2} - 19{,}1088902 - 8 = 42{,}461268$ is the length (in the middle) of one of the diagonal walks.

Then $42{,}461268 \times 8 \times 4 - 64 = 1294{,}760576$ is the area of the four walks in the diagonals.

Now $\overline{19{,}1088902+8}\,|^{2} \times 3{,}3137084 = 2435{,}2175541$ is the area of the octagonal walk and baſon.

And $2435{,}2175541 - 1210 = 1225{,}2175541$ is the area of that walk.

Then 3226,685589 is the area of all the gravel walks.

Hence the expence will be 215 £. 2 *s.* 3 *d.*

PART II.

Of ſolid MEASUREMENT.

Wherein will be ſhewn,

Firſt, *Definitions of the more common ſolids.*

Secondly, *Methods, of meaſuring, or of cubing of timber.*

Thirdly, *Methods of calculating the ſolidities and ſuperficies of various ſorts of ſolid figures.*

SECTION I.

DEFINITIONS.

I. A Solid is a figure contained under three dimenſions, *viz.* length, breadth, and depth, or thickneſs.

II. A ſolid whoſe baſes or ends are equal, parallel and like rectiline plane figures, and whoſe ſides are parallelograms, is called a *priſm*; and is denominated from the number of the ſides of its baſe.

III. If the ends and ſides are equal ſquares, the ſolid is call'd a *cube*. *Plate* 2. *Fig.* 1.

IV. If the baſe or end is a rectangle; the ſolid is call'd a *parallelopiped*. *Fig.* 2.

* V. If the baſe or ends are circles, the ſolid is called a *cylinder*. *Fig.* 3.

VI. A ſolid form'd on a plane rectiline baſe, having on its ſides right-lined plane triangles, whoſe vertices all meet in one and the ſame point; is called a *pyramid*; and is denominated from the number of the ſides of its baſe. *Fig.* 4.

* VII. If the baſe is a circle, the ſolid is call'd a cone. *Fig.* 5.

VIII. That ſolid which is terminated by a convex ſurface, whereof every point is equally diſtant from a certain point within the ſolid, is call'd a *ſphere*.——The right line paſſing thro' that point, equally diſtant from the ſurface, is call'd the *diameter* or *axis*. *Fig.* 6.

IX. In a pyramid, cone, ſphere, or any other tapering ſolid, a part thereof contain'd between two parallel ends, is call'd a *fruſtum:* And the parts wanting, at the ends of a *fruſtrum*, to compleat the tapering ſolid, are called *ſegments*.

X. If a fruſtum of a tapering ſolid be cut by a plane diagonally, from the extremity of one ſide at the leſſer end, to the extremity of the oppoſite ſide at the other end, each of theſe pieces is called a *hoof*, or an *ungula*; that being the greateſt which has the greateſt baſe.

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* *Note*, In Def. V. and VII. the number of ſides in the baſe or end, is ſuppoſed to be exceeding great, or infinite, and ſuch figures may be taken for curves.

SECTION II.

Of the meaſuring of timber trees.

WHEN trees are to be meaſured, their length is denoted by feet and inches; or by feet and tenth parts: And their circumference is taken by a ſmall cord, or chalk line; and this meaſure is denoted by the denominations that the length was taken in.

Workmen, and meaſurers, always divide their compaſs or circumference by 4; and this $\frac{1}{4}$ part, they call the *girt*, and eſtimate it as though it was the ſide of a ſquare, whoſe area was equal to a ſection of the tree, at the place it was girted.

It is a cuſtomary allowance to the buyer, to take the girt any where he pleaſes, between the greater end and the middle of the tree.

There are ſeveral other articles that enter into the meaſuring of timber for ſale, but they will be beſt underſtood by treating of them as particular caſes of the following propoſition.

PROPOSITION I.

To compute the ſolidity of round timber, by having the length and circumference given.

CASE I. *When the tree is ſtrait, and the ends are equal; or nearly ſo.*

Rule. Multiply the ſquare of one fourth of the circumference, by the length, and the product is the ſolidity, or (as commonly call'd), the content.

Examples.

I. *What is the ſolid content of a tree, whoſe compaſs is 32 inches, and the length 9 feet?*

Now $\frac{32}{4} = 8$ inches.

By decimals.

$$\begin{array}{rl} \text{8 inches} = 0{,}\cancel{6} & \text{feet} \\ \text{mult. by } \underline{0{,}\cancel{6}} & \\ 9)\underline{40} & \\ {,}4\cancel{4} & \\ \underline{\times\ 9} & \text{the length.} \\ \underline{4{,}00} & = \text{4 ſolid feet.} \end{array}$$

By duodecimals.

$$\begin{array}{rl} \text{f. i.} & \\ 0.8 & \rbrace\ \text{mult.} \\ \underline{0.8} & \\ 0.5.4 & \\ \underline{\times 9} & \\ \underline{4.0.0} & \end{array}$$

II. *What is the ſolidity of a tree, whoſe length is 31 feet, and the $\frac{1}{4}$ of the compaſs is 16 inches?*

By decimals.

$$\begin{array}{rl} \text{16 inc.} = 1{,}\cancel{3} & \text{feet} \\ \times\ \underline{1{,}\cancel{3}} & \\ 4\,\cancel{4} & \\ \underline{1\,\cancel{3}\,3} & \\ 1\,\cancel{7}\,7 & \\ \times\ \underline{3\,1} & \text{the length} \\ 1\,7\,7 & \\ \underline{5\,3\,3\,3} & \\ \underline{5\,5{,}1\,1} & \text{feet the ſolidity.} \end{array}$$

By duodecimals.

$$\begin{array}{rl} & \text{f. i.} \\ \text{16 inc.} = & 1.\ 4\ \rbrace\ \times \\ & \underline{1.\ 4} \\ & 0.\ 5.\ 4 \\ & \underline{1.\ 4.} \\ & 1.\ 9.\ 4 \\ & \underline{31} \\ & 0.10.\ 4 \\ & 23.\ 3 \\ & \underline{31} \\ & \underline{55.\ 1.\ 4} \end{array}$$

III. *What is the ſolidity of a tree, whoſe ¼ compaſs is 11 inches; and the length 40½ feet?*

By decimals.	By duodecimals.
11 inc. = 0,916 feet	f. i.
× 0,916	0 . 11
9)5500	0 . 11 } ×
6111	0 . 10 . 1
9166	× 40 . 6 . = length.
8250	0 . 5 . 0 . 6
,84027	0 . 3 . 4
× 40,5 the length	33 . 4
420138	34 . 0 . 4 . 6
33611111	
34,03125 feet, the ſolidity.	

IV. *Suppoſe a piece of timber is 9¾ feet long; and ¼ of the circumference is 39 inches: What is the ſolidity?*

By decimals.	By duodecimals.
39 inch. = 3,25 feet	f. i.
× 3,25	39 inch. = 3 . 3
1625	× 3 . 3
650	0 . 9 . 9
975	9 . 9
10,5625	10 . 6 . 9
length = 57,9 inverted	× by 9 . 9 = len.
95062	7 . 11 . 0 . 9
7394	95 . 0 .
528	102 . 11 . 9 . 9
102,984 feet, the ſolidity.	

V. *A tree whoſe* $\frac{1}{4}$ *of the compaſs is* 31 *inches, and the length* 24 *feet: What is the ſolidity?*

By decimals.

31 inc. = 2,58$\dot{3}$ feet
× $\dot{3}$85,2 inverted.
5167
1292
206
8
6,673
× 24 the length
26692
13346
160,152 feet the ſolidity.

By duodecimals.

f. i.
31 inc. = 2.7
× 2.7
1.6.1
5.2
6.8.1
× 24
0.2.0
16.0
144
160.2.0

Note, If the tree is crooked, its length muſt not be meaſured on either the convex, or concave ſide of the curve.

CASE II. *When the tree is taper, or unequally thick.*

RULE. Gird the tree in as many places as are thought neceſſary: the ſum of the ſeveral girds divided by their number, gives (as thought by workmen,) the mean compaſs; and the fourth of the mean compaſs ſquar'd and multiplied by the length, gives the ſolidity.

EX. VI. *A tapering tree* 15 *feet long, is girded in four places; in the firſt, it is* 5 *f.* 9 *in.; the ſecond,* 4 *f.* 5 *in.; the third,* 4 *f.* 9 *in.; and the fourth, is* 3 *f.* 9 *in.; what is the ſolidity?*

The

f. i.
The 1ſt compaſs 5·9
2d - - - 4·5
3d - - - 4·9
4th - - - 3·9
the number of girds = 4)18 . 8(
4 . 8 the mean compaſs.

f. i.
And $\frac{4\cdot 8}{4}$ = 1 . 2 . the quarter compaſs.

f. i.
1.2
× 1.2
0.2.4
1.2
1.4.4
× 15 = length.
20.5.0 the ſolidity.

CASE III. Branches or boughs meaſuring two feet in compaſs; (or 6 inches girt) are reckon'd as timber, and their ſolidity is to be computed and added to that of the tree.

Note, So much of the trunk, as meaſures leſs than 2 feet compaſs, is not eſteemed timber.

Ex. VII. *Required the ſolidity of a tree* 37 *feet long*; *and* 22 *inches, quarter compaſs*; *one branch* 12 *feet long*, *by* 28 *inches circumf. and another*, 8 *feet long*, *by* 24 *inches compaſs?*

Tree

Tree	branch	branch
1.10	$\frac{28}{4}$ = 0.7	$\frac{24}{4}$ = 0.6
× 1.10	× 0.7	× 0.6
1. 6.4	0.4.1	0.3
1.10	× 12	× 8
3. 4.4	4.1	2.0
× 37		
1. 0.4	Tree =	124.4.4
12. 4	one branch =	4.1.0
111	other branch =	2.0.0
124. 4.4	ſolidity =	130.5.4

CASE IV. *When the trees have their bark on.*

In meaſuring ſuch timber for ſale, 'tis common to make an allowance to the buyer, on account of the bark; thus, in oak, $\frac{1}{10}$ or $\frac{1}{12}$ part of the circcumference is deducted; but the allowance for the bark of elm, beach, aſh, &c. is leſs: This deduction being made, is ſuppoſed, to reduce the compaſs, to that which the tree will have, when the bark is ſtripped of. Therefore,

RULE. From the given circumference, ſubduct the allowance for bark; and with the remaining compaſs, find the ſolidity, as before.

EX. VIII. *An oak tree is* 45 *f.* 7 *inches long, and* 3 *f.* 8 *inches quarter compaſs; required the ſolid content; allowing* $\frac{1}{12}$ *for bark?*

```
        f.  i.
 12)  3 . 8
          3 . 8 = allowance
      3 . 4 . 4 = reduced quarter compaſs
    × 3 . 4 . 4
      0 . 1 . 1 . 5 . 4
      1 . 1 . 5 . 4
     10 . 1 . 0
     11 . 3 . 6 . 9 . 4
    × 45 . 7          = length
      6 . 7 . 0 . 11 . 5 . 4
      0 . 0 . 1 . 3 . 0
      0 . 2 . 9 . 9
      1 . 10 . 6
     11 . 3
    495
feet 514 . 11 . 5 . 11 . 5 . 4 the ſolidity.
```

CASE V. *When timber is to be reduced to loads.*

Meaſurers and workmen, reckon 40 feet of *unhewn*, or *rough timber*, and 50 feet of *hewn timber*, to a load, ſuppos'd to weigh a ton, or 20 hundred weight: For, ſay they; *hewn timber* is meaſured by the ſquare, and is very near exact; but *rough timber*, by the *girt*, (or quarter compaſs,) which is about $\frac{1}{5}$ leſs than exact; therefore, in the buying of timber, it amounts to much the ſame, whether it be meaſured by the *girt*, at 40 feet ſolid to a load; or meaſured exact, at 50 feet to a load; hence theſe

RULES Divide the feet, in rough timber, by 40, gives the Loads.
Divide the feet, in hewn timber, by 50, gives the Loads.

Ex. IX. *How many loads of timber is in that rough tree; whoſe length is* 28 *feet* 6 *inches, and the quarter compaſs,* 2 *feet* 9 *inches.*

$$\begin{array}{rl}
\text{f. i.} & \\
2.9 & \\
\times\ \underline{2.9} & \\
2.0.9 & \\
\underline{5.6} & \\
7.6.9 & \\
\times\ \underline{28.6} & \text{the length} \\
3.9.4.6 & \\
1.9.0 & \\
14.0 & \\
\underline{196} & \\
\underline{40)215.6.4.6} & \text{the ſolid content.} \\
5\frac{3}{8} & \text{loads.}
\end{array}$$

In the foregoing examples, are contain'd all the varieties that occur in the meaſuring of rough timber for ſale: But when timber is regularly and ſmoothly hewn; the ſolidities of ſuch pieces had beſt be computed by the rules given for *priſms*, *pyramids*, *cones*, &c. and their *fruſtrums*; as will be ſhewn hereafter.

Although the foregoing method of computing the ſolidity by the fourth part of the circumference, is commonly uſed by artificers, on account of its eaſe, yet in fact, it is very erroneous; for the fourth part of the circumference of a circle, cannot be equal to the ſide of a ſquare of equal area to that circle. Thus, if the circumference of a circle be

1, the ſide of a ſquare of equal area to that circle, is 0,2821; whereas by the falſe method of the *girt*, it is but 0,25.

Now the ſolidity of a round tree, may be found near enough, by either of the following rules.

I. Multiply the ſquare of the tree's compaſs by the length; and this product, by 0,07958, will give the ſolidity.

II. Multiply the ſquare of the tree's compaſs by $\frac{1}{12}$ of the length; divide this product by 24; to the quotient add a tenth of itſelf; this ſum, ſubtracted from the former product, leaves the ſolidity very near.

Note, inſtead of dividing by 24; it will be more convenient to divide by 6, and the quotient, by 4.

III. Find the ſolidity by the common method of the girt; under this, write its $\frac{1}{5}$ part; $\frac{1}{3}$ of this $\frac{1}{5}$ part; $\frac{1}{10}$ of this $\frac{1}{3}$ part; and the ſum of theſe four lines will be the ſolidity very nearly.

The two latter rules are beſt adapted for duodecimals, and the firſt for decimals.

The ſecond of the foregoing examples, is here wrought by each rule, both by decimals, and duodecimals.

The length 31 feet, and $\frac{1}{4}$ of the compaſs 16 inches; required the ſolidity.

Firſt. By decimals.

The compaſs = (16 × 4 =) 64 inches = 5,3 feet.

By rule 1ſt.

5,3
× 5,3
177
2666
28,4
× 31 = length
284
8533
881,777
× 85970,0
61724
7936
441
70
70,171

By rule 2d.

2,844 = ſquare of compaſs
$\frac{31}{12}$ feet = 2,583 = $\frac{1}{12}$ of length
9481
227555
1422222
5688888
24=6×4)73,48147 (12,2469 = $\frac{1}{6}$
3,0617 = $\frac{1}{4}$ of $\frac{1}{6}$ } add
3061 = $\frac{1}{10}$ of $\frac{1}{24}$ } add
3,3678 = ſum
− 3,3678
70,1136

By rule 3d.

55,11 = ſolidity by the girt.
11,022 = $\frac{1}{5}$
3,674 = $\frac{1}{3}$ of $\frac{1}{5}$
0,367 = $\frac{1}{10}$ of $\frac{1}{3}$
70,173

Secondly. By duodecimals.

By rule 2d.

```
                  f.  i.
                  5 . 4 = compaſs.
                  5 . 4
                  ---------
                  1 . 9 . 4
                 26 . 8
                 ---------
                 28 . 5 . 4
 31/12 feet =     2 . 7       = 1/12 length.
                 -----------
                 16 . 7 . 1 . 4
                 56 . 10 . 8
                 --------------
24=6×4)73 . 5 . 9 . 4(12 . 2 . 11 . 6 . 8 = 1/6.
   ſubtract 3 . 4 . 4 . 2   3 . 0 . 8 . 10 . 8 = 1/4 of 1/6.
            ----------
the ſolidity=70 . 1 . 5 . 2   3 . 7 . 3 . 3 . 3 = 1/10 of 1/24
                              ------------------
                              3 . 4 . 4 . 1 . 11
```

By rule 3d.

```
 f.   '   ''
55 . 1 . 4 by ex. 2d.
11 . 0 . 3 = 1/5
 3 . 8 . 1 = 1/3 of 1/5
 0 . 4   5 = 1/10 of 1/5
-----------
70 . 2 . 5 the ſolid content.
```

By each of theſe operations, the reſult is about $70\frac{1}{5}$ ſolid feet; but by the common method, the ſolidity is found to be only $55\frac{1}{10}$ feet; making a difference of above 15 feet, which is too conſiderable to be neglected.

Ex. XI. *Suppoſe the length of a tree, be* $36\frac{3}{4}$ *feet, and the compaſs* 87 *inches: What is the ſolid content?*

By decimals.

87 inches = 7,25 feet. Then by rule 1.

7,25 × 7,25 × 36,75 × 0,07958 = 153,722 the ſolidity.

By duodecimals.

```
              f.  i.
By rule 2d.   7 · 3          = 87 inches
              7 · 3
             ---------
              1 · 9 · 9
             50 · 9
             ---------
             52 · 6 · 9
36¼/12 feet = 3 · 0 · 9 = 1/12 of the length.
             -----------
              3 · 3 · 5 · 0 · 9
            157 · 8 · 3
            ---------------
24=12×2)160 · 11 · 8(13 · 4 · 11 · 8 = 1/12.
 ſubtract  7 ·  3 · 9  6 · 8 ·  5 · 10 = 1/2 of 1/12.
           ---------
the ſolid.=153 · 7 · 11    7 ·  3 · 3  = 1/6 of 1/24.
           ---------   ---------------
                       7 · 3 ·  9 · 1
```

By rule 3d.

f. ′ ″
$\frac{87}{4} = 21\frac{3}{4}$ inches = 1 . 9 . 9
1 . 9 . 9
1 . 4 . 3 . 9
1 . 4 . 3 . 9
1 . 9 . 9
3 . 3 . 5
× 36 . 9 = length
2 . 5 . 6 . 9
1 . 3 . 0
9 . 0
108
the ſolidity 120 . 8 . 6 . 9 by the girt.

$$\text{the correction}\left\{\begin{array}{r} 24 . 1 . 8 . 6 = \frac{1}{5} \\ 8 . 0 . 6 . 10 = \frac{1}{3} \text{ of } \frac{1}{5} \\ 9 . 7 . 3 = \frac{1}{10} \text{ of } \frac{1}{3} \end{array}\right.$$

the ſolidity 153 . 8 . 5 . 4

It has been already obſerv'd, that in multiplying feet, inches, and parts, by feet, inches, and parts, the ſuperficies expreſs'd by the product, may conſiſt of five denominations; among which, are two names beſide ſquare feet, ſquare inches, and ſquare parts: So in cubing of linear dimenſions, the ſolid expreſſed by the product, may have ſeven different places; among theſe are four terms, beſide cubic feet, cubic inches, and cubic parts; that is, two places between cubic feet, and cubic inches; and two places between cubick inches, and cubic parts.

To reduce, to cubic inches, the names between its place, and that of feet.

Rule Multiply the left-hand place by 12; to the product add its right-hand place; and 12 times this ſum will be cubic inches.

The ſame rule will ſerve, to reduce to cubic parts, the names between its place, and that of cubic inches.

Ex. *Suppoſe the ſolidity of a body was expreſſed by* 10ſ.. $8' . 7'' . 9''' . 5^{iv} . 11^{v} . 4^{vi}$ *required the equivalent cubic feet, inches and parts?*

Here the place of thirds, is cubic inches; and that of ſixths, is cubic parts.

Therefore $8' . 7'' . 9''' . = 103'' . 9''' = 1245'''$.

And $5^{iv} . 11^{v} . 4^{vi} = 71^{v} . 14^{vi} = 866^{vi}$.

Conſequently 10c.f. . 1245c.i. . 866c.p. = 10f. . $8' . 7'' . 9''' . 5^{iv} . 11^{v} . 4^{vi}$.

Here follows an explanation of the ſeveral names ariſing from the cubing of the linear dimenſions of ſolids.

1. The feet, are cubic feet.
2. The primes, are parallelopipeds of a foot long, a foot wide, and an inch thick.
3. The ſeconds, are parallelopipeds, either of a foot long, a foot broad, and a part thick. Or, of a foot long, an inch broad, and an inch thick.
4. The thirds are, either, parallelopipeds of a foot long, an inch wide, and a part thick; or, are cubic inches.
5. The fourths, are parallelopipeds; of a foot long, a part broad, and a part thick; or, of an inch long, an inch broad, and a part thick.
6. The fifths, are parallelopipeds, of an inch long, a part wide, and a part thick.
7. The ſixths, are cubic parts.

SECTION III.

Of divers ſolids, bounded by right-lin'd *and* circular figures.

PROPOSITION II.

Given the linear dimenſions of a cube, parallelopiped, cylinder, or of any priſm. To find the ſolidity.

RULE.

Multiply the area of the baſe, or end, by the length, or height, and the product will give the ſolidity.

EXAMPLE I.

What is the ſolidity of a cube, whoſe ſide is 12 *inches?*

Now 12 × 12 = 144, the area of the end.
And 144 × 12 = 1728, the inches in a foot.

EXAMPLE II.

What is the ſolidity of a block of marble, whoſe length (AB) *is* 10 *feet, breadth* (AC) $5\frac{3}{4}$ *feet, and depth* (AD) $3\frac{1}{2}$ *feet?* Pl. 2. Fig. 2.

Now 5,75 × 3,5 = 20,125, the area of the end.

And 20,125 × 10 = 201,25 feet, the ſolidity.

EXAMPLE III.

What is the ſolidity of a triangular priſm, whoſe length is 18 *feet, and one ſide of the quilateral end is* $1\frac{1}{2}$ *feet?*

The area of an equilateral triangle, whoſe ſide is 1, is 0,433013. (by tab. I. p. 144.)

Therefore, 1,5×1,5×0,433013=0,97427925, the area of the end.

Then, 0,97427925 × 18 = 17,5370265, the ſolidity.

EXAMPLE IV.

What is the ſolidity of a cylinder, whoſe length (AB) *is five feet, and the diameter* (AC) *of the end is* 2 *feet?* Fig. 3.

Now 2 × 2×0,7854=3,1416, the area of the end.

And 3,1416 × 5 = 15,708 feet, the ſolidity.

PROPOSITION III.

To find the ſolidity of a right pyramid, the meaſure of its baſe, or greater end; and the perpendicular height, or diſtance of the ends being given.

RULE.

Multiply the area of the baſe, or end, by a third part of the altitude, or length; and the product is the ſolidity.

EXAMPLE I.

What is the ſolidity of a pyramid, whoſe height (AC) *is* 24 *feet; and the ſide* (BD) *of its ſquare baſe is* 3 *feet?* Pl. II. Fig. 4.

Now $3 \times 3 = 9$, the area of the baſe.

And, $\frac{24}{3} = 8 = \frac{1}{3}$ of the height.

Then, $9 \times 8 = 72$, the ſolidity ſought.

EXAMPLE II.

What is the ſolidity of a pyramid whoſe height is 15 *feet, and one ſide of its hexagonal baſe is* 18 *inches?*

The area of a hexagon, whoſe ſide is 1, is found to be 2,598076. (by tab. I. p. 144.)

And $1,5 \times 1,5 \times 2,598076 = 5,845671$, the area of the baſe.

Then, $5,845671 \times \frac{15}{3} = 29,228355$, the ſolidity.

PROPOSITION IV.

To find the ſolidity of a right cone, the length, or perpendicular height, and the diameter, or circumference of the baſe being given.

RULE.

$$\text{Mult. the ſqu. of the}\left\{\begin{array}{l}\text{dia. by } 0{,}2618 \;=\; \dfrac{p}{12}\\[2ex] \text{circ. by } 0{,}026526 = \tfrac{1}{3}\text{ of }\dfrac{1}{4p}\end{array}\right\}$$

and the product by the height, gives the ſolidity.

EXAMPLE I.

What is the ſolidity of a cone, the diameter (AB) *of whoſe baſe is* 18 *inches; aud the altitude,* (CD) 15 *feet?* Pl. II. Fig. 5.

Now 18 inches = 1,5.
Then 1,5 × 1,5 × 0,2618 × 15 = 8,83575 feet the ſolid content.

EXAMPLE II.

If the circumference of the baſe of a cone be 40 *feet, and the height* 50 *feet: What is the ſolidity?*

Then 40 × 40 × 0,0265 × 50 = 2120 feet the ſolidity.

PROPOSITION V.

In a right cylinder, the length, and the diameter or circumference of the end being given; to find the convex ſuperficies.

RULE.

Multiply the number 3,1416 by the diameter, then the product multiplied by the length, will give the convex ſurface.

Or, Multiply the circumference by the length, and the product will be the convex ſuperficies.

EXAMPLE I.

What is the convex ſuperficies of a right cylinder, whoſe diameter (AC) *is* 30 *inches; and the length* (AB) 60 *inches?* Pl. II. Fig. 3.

Then (3,1416 × 30 × 60 =) 5654,88, the convex ſuperficies.

EXAMPLE II.

What is the ſuperficies of a cylinder whoſe circumference is 100 *inches, and the length* 14 *feet?*

Now 100 inches = 8.$\cancel{3}$ feet; and 8,$\cancel{3}$ × 14 = 116,$\cancel{6}$ feet, the curve ſuperficies.

And 8,$\cancel{3}$ × 8,$\cancel{3}$ × 0,07958 × 2 = 11,052$\cancel{7}$, the area of both the ends.

Then, 116,$\cancel{6}$ + 11,052$\cancel{7}$ = 127,719$\cancel{4}$, the whole ſuperficies.

PRO-

PROPOSITION VI.

To find the convex ſuperficies of a right cone; the $\left\{\begin{array}{l}\textit{diameter}\\ \textit{circumf.}\end{array}\right\}$ *of whoſe baſe or end, and the length of the ſlant ſide being given.*

RULE.

Multiply the $\left\{\begin{array}{l}\text{diam. by } 1{,}5708 \left(=\frac{p}{2}\right)\\ \text{circu. by } 0{,}5\end{array}\right\}$ and the product by the length of the ſide, gives the convex ſuperficies.

EXAMPLE I.

What is the convex ſuperficies of a right cone, the length of the ſlant ſide (AC) *being* 50; *and the diameter of the baſe* (AB) 20? Pl. II. Fig. 5.

Then (1,5708 × 20 × 50 =) 1570,8 is the convex ſurface.

EXAMPLE II.

What is the convex ſuperficies of a right cone, whoſe circumference at the baſe is 24 *feet, and the length of the ſlant ſide* 32 *feet?*

Then (24 × 0,5 × 32 =) 384 is the convex ſuperficies.

PROPOSITION VII.

In the fruſtum of a pyramid; whoſe ends are alike regular polygons, not exceeding 12 *ſides; to find the ſolidity* (S) *thereof; the linear meaſures of* (AB,ab) *the two parallel ends, and their diſtance* (Cc) *being known.* Pl. II. Fig. 7.

Rule 1. Multiply one ſide of the greater end, by one ſide of the leſſer end.

2. To the product, add one third of the ſquare of the difference of thoſe ſides.

3 Multiply the ſum by the length.

4. Then this product multiplied by the polygon's factor (in tab. I. p. 144.) will give the ſolidity.

$$\text{Or } S = \overline{Ab \times ab \times \tfrac{1}{3}\,\overline{AB \quad ab}^{2}} \times Cc \times N.$$

EXAMPLE I.

What is the ſolidity of the fruſtum of a ſquare pyramid, one ſide of the greater end being 18 *inches; that of the leſſer end,* 15 *inches, and the height* 60 *inches?*

Now $18-15=3$ the difference of the ſides.

And $\frac{3\times3}{3}$ the third part of the ſquare of that difference.

Then $(\overline{18\times15+3}\times60=)16380$ is the ſolidity required.

EXAMPLE II.

What is the ſolidity of the fruſtum of a hexagonal pyramid; the ſide of whoſe greater end is 3 feet; that of the leſſer end, 2 feet; and the length 12 feet?

Now $3-2=1$, and $\frac{1\times1}{3}=0,3$ is $\frac{1}{3}$ of the ſquare of the difference of the ſides.

Then $\overline{3\times2+0,\acute{3}}\times12\times2,598076$ gives 197,453776 for the ſolidity required.

If the ends of a pyramid's fruſtum be of any other figure than that of a regular polygon; find the ſide of a ſquare, whoſe area ſhall be equal to the area of the greater end, and do the ſame for the leſſer end; then work as the foregoing rule directs.

PROPOSITION VIII.

To find the ſolidity (S) *of the fruſtum of a right cone, the length or height* (Cc), *and the* $\left\{\begin{array}{l}\textit{diameter (AB, ab)}\\ \textit{circumference}\end{array}\right\}$ *at each end being given.* Pl. II. Fig. 8.

RULE.

1. Multiply the $\left\{\begin{array}{l}\text{diam.}\\ \text{circu.}\end{array}\right\}$ at one end, by that at the other.

2. To the prod. add the ſqus. of thoſe $\left\{\begin{array}{l}\text{diameters.}\\ \text{circumſes.}\end{array}\right.$

3. Multiply the ſum by the given length.

4. The product multip. by the N°. $\left\{\begin{array}{l}0{,}2618\\ 0{,}026526\end{array}\right.$ will give the ſolidity.

Or $S = \overline{AB \times ab + \overline{AB}^2 + \overline{ab}^2} \times Cc \times \frac{p}{12}$.

EXAMPLE I.

What is the ſolidity of the fruſtum of a cone, the diameter of the greater end being 4 feet ; that of the leſſer end, 2 feet ; and the altitude 9 feet ?

Now 4×2=8, the product of the two diameters.
And 4×4=16 : 2×2=4 ; 8+16+4=28.
Then 28×9×0,2618=65,9736 feet, the ſolidity.

EXAMPLE II.

What is the ſolidity of the fruſtum of a cone; the circumference of the greater end being 40; *that of the leſſer end* 20; *and the length or height* 50?

Now $40\times20+40\times40+20\times20=2800$.

Then $(2800\times50\times0{,}026526=)3713{,}64$ is the lidity required.

Note, The Rule of Prop. VII. will find the ſolidity of a fruſtum of a cone, by uſing the diameters of the ends of the cone, inſtead of the ſides of the pyramid; and changing the 4th article, for that in the Rule of Prop. VIII.

And the Rule of Prop. VIII. will ſerve for Prop. VII. by making a like change.

By theſe two laſt Propoſitions, ſhould all unequal ſquared or round timber be meaſured; for moſt Trees, being bigger at the ground end than at the other, may be conſidered (when firſt cut down, and the branches lopt off) as the fruſtums of cones; and if the ſides are cut rectangular, they then may be conſidered as the fruſtums of pyramids; and conſequently, in either caſe, they ſhould be meaſured according to the figure they repreſent, ſuppoſing them to be regular; but if the difference of the ends be ſmall, there is no need of having recourſe to any other directions than thoſe given in the firſt propoſition; and as all pyramids and cones, are conſidered as having their ſides perfectly ſtrait, moſt trees will differ from them, on account of the inequality of their ſides or girths.

PRO-

PROPOSITION IX.

To find the convex ſurface of the fruſtum of a right cone, the length of the ſlant ſide, and the $\left\{\begin{array}{l}\textit{diameters}\\ \textit{circumf.}\end{array}\right\}$ *of the parallel ends being given.*

RULE.

Multiply the ſum of the $\left\{\begin{array}{l}\text{diameters by } 1{,}5708\\ \text{circumfe. by } 0{,}5\end{array}\right\}$ the product multiplied by the length of the ſide will give the ſolidity.

EXAMPLE I.

What is the convex ſurface of the fruſtum of a right cone; the diameters of the ends being 8 *and* 4 *feet; and the length of the ſide* 20 *feet?*

Now $8+4=12$ the ſum of the diameters.

And $12\times1{,}5708\times20=376{,}992$ the convex ſurface.

EXAMPLE II.

What is the convex ſurface of the fruſtum of a right cone; the circumference of the greater end being 30 *feet, that of the leſſer end* 10 *feet; and the length of the ſlant ſide* 20 *feet?*

Then $(\overline{30+10}\times0{,}5\times20=)$ 400 feet is the convex ſurface required.

PRO-

PROPOSITION X.

To find the length or height, (CD) *of a pyramid or cone, the linear dimenſions* (AB, ab, Cc) *of a fruſtum of that pyramid or cone being known.* Pl. II. Fig. 7. 8.

RULE.

Multiply the height of the fruſtum by the $\left\{\begin{matrix}\text{ſide}\\ \text{dia.}\end{matrix}\right\}$ of the greater end.

Divide the product by the diff. of the $\left\{\begin{matrix}\text{ſides}\\ \text{dias.}\end{matrix}\right\}$ of the two ends; and the quotient will be the height of the $\left\{\begin{matrix}\text{pyramid.}\\ \text{cone.}\end{matrix}\right.$

Or $CD = \frac{Cc \times AB}{AB \quad ab}$.

EXAMPLE I.

In a fruſtum of a ſquare pyramid; one ſide of the greater end is 5 *feet; one ſide of the leſſer end is* 3 *feet; and the diſtance of the ends is* 8 *feet: What is the height of that pyramid?*

Now $5-3=2$ the difference of the ſides.

Then $\left(\frac{5\times8}{2}=\right)$ 20 is the height of the pyramid.

EXAMPLE II.

There is a fruſtum of a cone, the diameter of the greater end is 83 *inches; that of the leſſer end is* 54 *inches; and the altitude is* 12 *feet, what is the altitude of the cone?*

Now 83 54=29 inches, which is 2,416 feet, the difference of the diameters; and 83 inches = 6,916 feet.

Then $\frac{6,916\times12}{2,416} = 34,3448$ is the height of the cone as required.

PRO-

PROPOSITION XI.

The dimenſions of a pyramid or cone being given; to find what length from the vertex will anſwer to a given part of the ſolidity?

RULE.

Say as the ſolidity of the whole pyramid or cone, is to the cube of its altitude; ſo is any given part of the ſolidity, to the cube of it's altitude, reckon'd from the vertex downwards; whoſe cube-root is the length.

EXAMPLE.

There is a conical piece of timber, the diameter of whoſe baſe is 18 *inches, and the length* 12 *feet: What diſtance from the vertex muſt the ſaw be applied, to cut it into two pieces of equal ſolidity.*

Now 18 inches = 1,5 feet.

And 1,5 × 1,5 × 12 × 0,2618 = 7,0686 feet the ſolidity; half thereof, is 3,5343.

And 12×12×12=1728 the cube of the length.

Then 7,0686 : 1728 : : 3,5343 : 864, whoſe cube root is 9,5244; and ſo far from the vertex, muſt the ſaw be applied.

PROPOSITION XII.

In a priſmoid, having given its length or height, and alſo the length and breadth of each end; to find the ſolid content.

Note, A priſmoid is a ſolid contain'd under ſix planes; whereof the parallel ends are unlike rectangles; and of the other four ſides, each oppoſite two, are equal trapeziums.

RULE.

Multiply the length at the greater end, by the breadth at the leſſer end; and the length at the leſſer end, by the breadth at the greater end.

To half the ſum of theſe two products, add the areas of the two ends.

The ſum multiplied by a third of the height, gives the ſolidity.

Or, To the breadth of the greater end, add half the breadth at the leſſer end, multiply the ſum by the length at the greater end.

To the leſſer breadth, add half the greater breadth, multiply the ſum by the leſſer length.

The ſum of theſe two products, multiplied by a third of the height, gives the ſolidity.

EXAMPLE.

What is the ſolid content of a priſmoid, whoſe greater end meaſures 12 *inches by* 8; *the leſſer end,* 8 *inches by* 6; *and the length, or height,* 60 *inches?*

Now $12\times6=72$; $8\times8=64$; and $\frac{72+64}{2}=68$.

Then $\overline{68+12\times8+8\times6}\times20=4240$, the ſolidity.

Or, $\overline{8+\frac{6}{2}}\times12=132$; and $\overline{6+\frac{8}{2}}\times8=80$.

Then $\overline{132+80}\times20=4240$ is the ſolidity.

PRO-

PROPOSITION XIII.

*To find the ſolidity of the Hoofs (aefb*AE,FB AE*fb) of the fruſtum of a pyramid, the linear meaſures of the end, and the length being known.* Pl. II. Fig. 7.

If it is of any other form than a ſquare pyramid, reduce it to ſuch, by finding the ſide of a ſquare of equal area; then

RULE.

To the ſquare of the ſide of one end, add one half the product of the ſides of the two ends; this ſum, multiply'd by one third of the height, gives the ſolidity.

Note, The ſolidity of the greater or leſſer ungula will be obtain'd, according to the end of the fruſtum uſed.

Or greater hoof $= \overline{\overline{AB}^2 + \frac{1}{2}\,\overline{AB} \times \overline{ab}} \times \frac{1}{3}\,Cc.$

leſſer hoof $= \overline{ab}^2 + \frac{1}{2}\,\overline{AB \times ab} \times \frac{1}{3}\,Cc.$

EXAMPLE

There is a fruſtum of a ſquare pyramid, one ſide of the greater end is 1,5 *feet; the ſide of the leſſer end is* 1,25 *feet, and the height is* 5 *feet; what is the ſolidity of the ungula?*

Now $1{,}5\times1{,}5=2{,}25$, the ſquare of the greater ſide.

And $\frac{1{,}5\times1{,}25}{2}=0{,}9375$.

Then $(\overline{2{,}25+0{,}9375}\times\frac{5}{3}=)5{,}3125$ is the ſolidity of the greater of the hoof.

Again, $\underline{1{,}25\times1{,}25=1{,}5625}$.

Then $(\overline{1{,}5625+0{,}9375}\times\frac{5}{3}=)\ 4{,}1\dot{6}$ is the ſolidity of the leſſer ungula.

PROPOSITION XIV.

In a conical fruſtum, the diameters, and diſtance of the two parallel ends being given; to find the ſolidity of each hoof, when the fruſtum is cut by a diagonal plane. (Fig. 8.)

For the greater hoof (S).

RULE.

From the cube of the greater diameter, take the ſquare root of the product of the cubes of the two diameters.

Divide the remainder by the difference of the two diameters.

Multiply the quotient by the height.

The product multiplied by 0,2618, will give the ſolidity.

$$\text{Or } S=\frac{\overline{AB}^3-\sqrt{\overline{AB}^3\times\overline{ab}^3}}{AB-ab}\times Cc\times\frac{p}{12}.$$

RULE.

For the leſſer hoof (s).

RULE.

From the ſquare root of the product of the cubes of the two diameters, take the cube of the leſſer diameter.

Divide the remainder by the difference of the two diameters.

Multiply the quotient by the height.

The product multiplied by 0,2618 will give the ſolidity.

$$\text{Or } s = \frac{\sqrt{\overline{AB}^3 \times \overline{ab}^3} - \overline{ab}|^3}{AB - ab} \times Cc \times \frac{p}{12}.$$

EXAMPLE.

There is a conical fruſtum, the diameter of the greater end is 4 feet; that of the leſſer 2 feet; and the height 9 feet; what is the ſolidity of each hoof?

Now $(4 \times 4 \times 4 =)$ 64 is the cube of the greater diameter.

And $(2 \times 2 \times 2 =)$ 8 is the cube of the leſſer diameter.

Alſo $64 \times 8 = 512$; whoſe ſquare root is 22,6272.

Then $\left(\frac{64 - 22{,}6272}{4 - 2} \times 9 \times 0{,}2618 =\right)$ 48,7413 is the ſolidity of the greater hoof.

And $\frac{22{,}6272 - 8}{4 - 2} \times 9 \times 0{,}2618 =)$ 17,2323 is the ſolidity of the leſſer hoof.

SECTION IV.

Of a ſphere and its parts.

PROPOSITION XV.

To find the ſuperficies of a ſphere or globe.

I. *The diameter being known.*

Rule. Multiply the ſquare of the diameter by 3,1416; ($=p$) and the product is the ſuperficies.

Ex. *What is the ſuperficies of a ſphere, whoſe diameter is $1\frac{1}{3}$ feet?*

Then $1{,}\cancel{3} \times 1{,}\cancel{3} \times 3{,}1416 = 5{,}5850\cancel{6}$ feet is the ſuperficies.

II. *The circumference of a circle biſecting that ſphere being known.*

Rule. Multiply the ſquare of the circumference by 0,31832; $\left(=\frac{1}{p}\right)$ and the product is the ſuperficies.

E x. *What is the ſuperficies of a ſphere, whoſe circumference is* 4,1888 *feet?*

Then (4,1888 × 4,1888 × 0,31832 =) 5,5852 feet is the ſuperficies.

III. *The diameter and circumference being known.*

R u l e. Multiply the diameter by the circumference, and the product is the ſuperficies.

E x. *What is the ſuperficies of a ſphere, whoſe diameter is* $1\frac{1}{3}$ *feet, and the circumference is* 4,1888 *feet?*

Then (1,3 × 4,1888 =) 5,58506 is the ſuperficies.

PROPOSITION XVI.

To find the ſolidity of a ſphere.

I. *The diameter being known.*

R u l e. Multiply the cube of the diameter by 0,5236 $\left(=\frac{p}{6}\right)$ and the product is the ſolidity.

E x. *What is the ſolidity of a ſphere, whoſe diameter is* $1\frac{1}{3}$ *feet?*

Then (1,3,×1,3×1,3×0,5236=)1,2411 feet is the ſolidity.

II. *The circumference of a circle biſecting that ſphere being known.*

Rule. Multiply the cube of the circumference by 0,016887; $\left(=\frac{1}{6pp}\right)$ and the product is the ſolidity.

Ex. *What is the ſolid content of a ſphere, whoſe circumference is* 4,1888?

Then (4,1888×4,1888×41888×0,016887=) 1,2411 is the ſolid content.

If the ſuperficies and diameter are known.

Rule. Multiply the ſuperficies by one ſixth of the diameter, and the product is the ſolidity.

Ex. *What is the ſolidity of a ſphere whoſe diameter is* $1\frac{1}{3}$ *feet, and the ſuperficies* 5,58506 *feet?*

Then $\left(5{,}58506 \times \frac{1{,}3}{6} =\right)$ 1,2411 feet is the ſolidity.

IV.

Ex. *What is the ſuperficies of a ſphere, whoſe circumference is* 4,1888 *feet?*

Then (4,1888 × 4,1888 × 0,31832 =) 5,5852 feet is the ſuperficies.

III. *The diameter and circumference being known.*

Rule. Multiply the diameter by the circumference, and the product is the ſuperficies.

Ex. *What is the ſuperficies of a ſphere, whoſe diameter is* $1\frac{1}{3}$ *feet, and the circumference is* 4,1888 *feet?*

Then ($1,\dot{3}$ × 4,1888 =) 5,58506 is the ſuperficies.

PROPOSITION XVI.

To find the ſolidity of a ſphere.

I. *The diameter being known.*

Rule. Multiply the cube of the diameter by 0,5236 $\left(=\frac{p}{6}\right)$ and the product is the ſolidity.

Ex. *What is the ſolidity of a ſphere, whoſe diameter is* $1\frac{1}{3}$ *feet?*

Then (1,3,×1,3×1,3×0,5236=)1,2411 feet is the ſolidity.

II. *The circumference of a circle biſecting that ſphere being known.*

Rule. Multiply the cube of the circumference by 0,016887; $\left(=\frac{1}{6pp}\right)$ and the product is the ſolidity.

Ex. *What is the ſolid content of a ſphere, whoſe circumference is* 4,1888?

Then (4,1888×4,1888×41888×0,016887=) 1,2411 is the ſolid content.

If the ſuperficies and diameter are known.

Rule. Multiply the ſuperficies by one ſixth of the diameter, and the product is the ſolidity.

Ex. *What is the ſolidity of a ſphere whoſe diameter is* $1\frac{1}{3}$ *feet, and the ſuperficies* 5,58505 *feet?*

Then $\left(5{,}58505 \times \frac{1{,}3}{6} =\right)$ 1,2411 feet is the ſolidity.

IV. *If the ſuperficies and circumference are known.*

RULE. Multiply the ſuperficies by the circumference, and the product multiply'd by 0,05305 $\left(=\frac{1}{6}p\right)$ gives the ſolidity.

Ex. *What is the ſolidity of a ſphere whoſe ſuperficies is* 5,58506, *and whoſe circumference is* 4,1888?

Then (5,58506×4,1888×0,05305=)1,2411 is the ſolidity.

Note, The ſolidity of a ſphere, is equal to two thirds of its circumſcribing cylinder.

PROPOSITION XVII.

The ſuperficies of a ſphere being known.

I. *To find the diameter.*

RULE. Multiply the ſquare root of the ſuperficies by 0,56419 $\left(=\sqrt{\frac{1}{p}}\right)$ and the product will be the diameter.

Ex. *What is the diameter of that ſphere, whoſe ſuperficies* 5,5852 *feet?*

Now $\sqrt{5,5852}$ = 2,3633.
Then (2,3633 × 0,56419 =) 1,3 feet is the diameter required.

II.

II. *To find the circumference of a circle biſecting that ſphere.*

Rule. Multiply the ſquare root of the ſuperficies by 1,77245, $(=\sqrt{p})$ and the product will be the circumference.

Ex. *If the ſuperficies is* 5,5852; *what is the circumference?*

Then $(\sqrt{5,5852} \times 1,77245 =)$ 4,1888 is the circumference.

III. *To find the ſolidity.*

Rule. Multiply the ſuperficies, its ſquare root, and 0,0940316 $\left(=\frac{1}{6}\sqrt{\frac{1}{p}}\right)$ continually, the product will give the ſolidity required.

Ex. *The ſuperficies* = 5,5852; *required the ſolidity?*

Then $(5,5852 \times \sqrt{5,5852} \times 0,0940316 =)$ 1,2411 is they ſolidity ſought.

PROPOSITION XVIII.

The ſolidity of a ſphere being known.

I. *To find the diameter.*

RULE. The cube root of the ſolidity, multiplied by 1,2407 $\left(=\sqrt[3]{\frac{6}{p}}\right)$ will give the diameter.

EX. *Required the diameter of that ſphere whoſe ſolidity is* 1,2411?

Now $\sqrt[3]{1,2411} = 1,074655$.
Then $(1,074655 \times 1,2407 =) 1,3$ is the diameter.

II. *To find the circumference of a circle biſecting that ſphere.*

RULE. Multiply the cube root of the ſolidity by 3,897777 $(= \sqrt[3]{6pp})$ the product will be the circumference.

EX. *If the ſolidity is* 1,2411; *required the circumference?*

Then $(\sqrt[3]{1,2411} \times 3,897777 =) 4,1888$ is the circumference ſought.

III. *To find the ſuperficies.*

Rule. The cube root of the ſolidity multiplied by itſelf, and the product by 4,835976 $(=\sqrt[3]{36p})$ will give the ſuperficies required.

Ex. *If the ſolidity is* 1,2411; *what is the ſuperficies?*

Now $\sqrt[3]{1,2411} = 1,074655$.
Then $(1,074655 \times 1,074655 \times 4,835976 =)$ 5,58498 is the ſuperficies ſought.

PROPOSITION XIX.

To find the convex ſuperficies (s) *off a ſegment of a ſphere cut of by a plane.*

I. *The diameter* (BA) *of the ſphere, and the height* (DE) *of the ſegment* (FEG) *being known.* Pl. II. Fig. 10.
Rule. Multiply the diameter of the ſphere by the height of the ſegment; then the product multiplied by 3,1416 will give the convex ſuperficies.

Or $s = AB \times DE \times p$.

Ex. *What is the convex ſuperficies of a ſegment whoſe height is* $4\frac{1}{2}$, *and cut from a ſphere of* 21 *inches diameter?*

Then $21 \times 4,5 \times 3,1416 =)$ 296,8812, is the convex ſuperficies.

II. *The diameter* (FG) *of the baſe of the ſegment, and its height* (DE) *being known.*

RULE. To the ſquare of the diameter of the baſe, add the ſquare of twice the height; the ſum multiplied by 0,7854 will give the ſuperficies required.

Or $s = \overline{\overline{FG}^2 + \overline{2\,DE}^2} \times \frac{p}{4}$.

Ex. *What is the convex ſuperficies of that ſpherical ſegment, the diameter of whoſe baſe is* 17,23368, *and whoſe height is* 4,5?

Now $17{,}23368 \times 17{,}23368 = 296{,}999$, &c. or 297.

And $\overline{4{,}5 \times 2}|^2 = 81$.

Then ($\overline{297 + 81} \times 0{,}7854 =$) 296,8812 is the ſuperficies required.

PROPOSITION XX.

To find the ſolidity (S) *of a ſegment of a ſphere.*

I. *The diameter* (FG) *of the baſe of the ſegment, and its height* (ED) *being known.* Pl. II. Fig. 10.

RULE. To thrice the ſquare of (FD) half the diameter of the baſe, add the ſquare of the height (DE), multiply the ſum by the height; then the product multiplied by 0,5236 will give the ſolidity.

Or $(S) = \overline{3\,\overline{DF}^2 + \overline{DE}^2} \times DE \times \frac{p}{6}$.

Ex. *What is the ſolidity of a ſpherical ſegment, the diameter of the baſe being* 17,23368 *inches; and it's height* 4,5?

Now $\frac{17,23368}{2} \times \frac{17,23368}{2} \times 3 = 222,75.$

And $4,5 \times 4,5 = 20,25.$

Then $(222,75 + 20,25 \times 4,5 \times 0,5236 =)$ 572,5566 is the ſolidity required.

II. *The diameter* (AB) *of the ſphere, and the height* (DE) *of the ſegment being known.*

Rule. From thrice the diameter, ſubtract twice the height; multiply the remainder by the the ſquare of the height; the product multiplied by 0,5236 will give the ſolidity.

$$\text{Or } S = \overline{3\,AB - 2\,DE} \times \overline{DE}^2 \times \frac{p}{6}.$$

Ex. *In a ſphere whoſe diameter is* 21; *what is the ſolidity of a ſegment thereof, whoſe height is* 4,5?

Now $\overline{21 \times 3} - \overline{4,5 \times 2} = 54.$

And $4,5 \times 4,5 = 20,25.$

Then $(54 \times 20,25 \times 0,5236 =)$ 572,5566 is the ſolidity ſought.

PROPOSITION XXI.

To find the convex ſuperficies (s) of a ſpherical zone or fruſtum.

I. *The diameter* (AB=*d*) *of the ſphere, and the breadth* (MN=*b*) *of the zone, or diſtance of the parallel ends being known.* Pl. II. Fig. 10.

Rule. Multiply the diameter of the ſphere by the breadth of the zone; the product multiplied by 3,1416 (=*p*) will give the ſuperficies required.

Or $s = dbp$.

Ex. *What is the convex ſurface of a ſpherical zone whoſe breadth is 4 inches; and cut from a ſphere of 25 inches diameter?*

Then 25 × 4 × 3,1416 = 314,16 the convex ſurface.

II. *Having the diameters* (HI=*c*, KL=*b*) *of the ends, and their diſtance* (MN=*h*) *given.*

Rule. Find the diameter of the ſphere (by Prop. XXIII. Part I.) and then find the convex ſurface by the former rule.

Ex. I. *In a ſpherical zone, the diſtance of whoſe parallel ends is 4 inches, the diameter of the greater end 24 inches, and that of the leſſer end, 20 inches: What is the convex ſurface, when the centre of the ſphere is without the zone?*

Now $\dfrac{\frac{24}{2}\times\frac{24}{2}-\frac{20}{2}\times\frac{20}{2}+4\times4}{2\times4}=3,5$ the diſtance of the greater end from the centre.

And $3,5\times2=7$.

Alſo $24\times24+7\times7=625$. whoſe ſquare root is 25.

Then $25\times4\times3,1416=314,16$ the convex ſurface.

Ex. II. *What is the convex ſurface of a ſpherical zone, the diſtance of whoſe parallel ends is 11 inches; the diameter of the greater end 24, and that of the leſſer end 20; the centre of the ſphere lying between the ends.*

Now $\dfrac{\frac{20}{2}\times\frac{20}{2}+11\times11-\frac{24}{2}\times\frac{24}{3}}{11\times2}=3,5$ the diſtance of the greater end from the center.

And $3,5\times2=7$.

Alſo $24\times24+7\times7=625$ whoſe ſquare root is 25, the diameter of the ſphere.

Then $25\times11\times3,1416=863,94$ ſquare inches, the convex ſurface.

III. *When the diameter* (HG$=c$) *and height* (DE$=v$) *of a ſegment cut from a hemiſphere, is given; the convex ſurface* (s) *of the remainder* (HIBA) *may be thus found.* Pl. II. Fig. 10.

RULE.

1. Divide the fourth power of the diameter by thirty-two times the ſquare of the height.

2. From the quotient, take half the ſquare of the height.

3. The remainder multiplied by 3,1416 gives the convex ſurface.

Or $$s = \overline{\frac{c^4}{32\,vv} - \frac{vv}{2}} \times p.$$

Ex. *From a hemiſphere, whoſe diameter is unknown, there is cut a ſegment, wherein the diameter of the baſe is 20 inches; and the height, or verſed ſine, is 5 inches: Required the convex ſuperficies of the remaining part?*

Now $$\frac{20 \times 20 \times 20 \times 20}{32 \times 5 \times 5} = 200.$$

And $$200 - \frac{5 \times 5}{2} = 187{,}5.$$

Then $187{,}5 \times 3{,}1416 = 589{,}05$ the convex ſurface required.

PROPOSITION XXII.

To find the ſolidity (S) *of a ſpherical zone, the radius* (MI=c,NL=b) *of each end, and their diſtance* (MN=h) *being known.*

Rule. To the ſquare of the two radius's add one third of the ſquare of the height: The ſum multiplied by the height, and the product by 1,5708, gives the ſolidity.

$$\text{Or } S=\overline{cc+bb+\tfrac{1}{3}hh}\times h\times\frac{p}{2}.$$

Ex. *What is the ſolid content of a zone, whoſe greater diameter is* 24 *inches, that of the leſſer* 20 *inches, and the height or diſtance of the ends is* 4 *inches?*

$$\text{Now } \left.\overline{\frac{24}{2}}\right|^2+\left.\overline{\frac{20}{2}}\right|^2+\frac{4\times4}{3}=242,\dot{3}.$$

Then $249,\dot{3}\times4\times1,5708=1566,6112$ the ſolidity.

DEFINITION.

A *circular ſpindle*, is a ſolid deſcribed by the rotation of a circular ſegment about its chord: The circle, of which the ſegment is a part, call, the *prime circle*; and the diſtance of its centre from that of the (middle of the chord, or) centre of the ſpindle, call the *central diſtance.*

PROPOSITION. XXIII.

In the fruſtum (EFHG) *of a circular ſpindle* (AFBE); *wherein, the diameter* (FE=2D) *paſſing thro' the centre* (D), *a diameter* (HG=2*d*) *in another place, and the diſtance* (DI=*b*) *of thoſe diameters are known; to find the central diſtance* (CD=*x*). Pl. II. Fig. 11.

RULE.

The difference of the ſquares of the half diameters, taken from the ſquare of their diſttance, and the remainder divided by the difference of the diameters, gives the central diſtance.

$$\text{Or } x = \frac{bb - \overline{DD - dd}}{D - d}.$$

The central diſtance, added to half the greater diameter, gives the radius of the prime circle.

Ex. *In the fruſtum of a circular ſpindle, whoſe greateſt diameter* EF=36, *leſſer* GH=16; *and their diſtance* DI=20: *Required the central diſtance* CD, *and radius* CE *of the prime circle?*

Now $\left(\overline{20}\big|^2 - \overline{\overline{\frac{36}{2}}\Big|^2 - \overline{\frac{16}{2}}\Big|^2} \div \overline{36 - 16} =\right)$ 7 is the central diſtance.

And $\left(\frac{36}{2} + 7 =\right)$ 25 is the radius of the prime circle.

PRO-

PROPOSITION XXIV.

To find the ſolidity (S) *of a circular ſpindle; its length* ($AB=l$), *and greater diameter* ($EF=D$) *being known.*

RULE.

1ſt, Find the radius ($CE=r$) of the prime circle, and central diſtance ($CD=x$). (Obſerving, that in this caſe, the leſſer diameter $=0$).

2d, Find the area (a) of a circular zone, whoſe breadth (AB) is equal to the length of the ſpindle; and the chord ($=2CD$) of each equal end, is equal to twice the central diſtance.

3d, Multiply the zone by the central diſtance, call the product A.

4th, To the ſquare of the radius, add twice the ſquare of the central diſtance, multiply the ſum by two thirds of the length, call the product B.

5th, Then A taken from B, and the remainder multiplied by 3,1416, will give the ſolidity.

Or $\overline{\overline{rr+2xx}\times\frac{2}{3}l-ax}\times p=S$.

EXAMPLE.

What is the ſolidity of a circular ſpindle, whoſe length is 48; and the greater diameter is 36?

Now $\left(\overline{\frac{48}{2}}\Big|^2 - \overline{\frac{36}{2}}\Big|^2 \div 36 =\right)$ 7 is the central diſtance.

And $\left(\frac{36}{2}+7=\right)$ 25 is the radius of the prime circle.

Alſo 636,611238 is the area of the generating circular ſegment.

Then $\left(\frac{\overline{7\times 2}+\overline{7\times 2}}{2}\times 48+\overline{636{,}611238}\times 2=\right)$ 1945,222476 is the area of the zone.

And $(1945{,}222476\times 7=)13616{,}557332=A$.

Alſo $(\overline{\overline{25}|^2+2\times\overline{7}|^2}\times\frac{2}{3}\times 48=)23136=B$.

Conſequently $(B-A=)9519{,}462668\times 3{,}1416=)$ 29906,2792 is the ſolidity required.

PROPOSITION XXV.

To find the ſolidity, (S) of a fruſtum, of a circular ſpindle, wherein, one end (EF) paſſes thro' the center (D): The diameters (EF,GH) of thoſe ends, and their diſtance being known.

RULE.

Find the central diſtance ($CD = x$), and the radius ($CE = r$) of the prime circle.

Find the area (a) of a circular zone, wherein, one end, (or chord,) is equal to the fruſtum's leſſer diameter added to twice the central diſtrance (or $= 2\ CD + HG$); the other end, equal to the diameter of the prime circle; and the breadth, equal to the given diſtance (DI) of the diameters; multiply this zone by the central diſtance, call the product A. (Or put $A = xa$.)

To the ſquare of the radius of the prime circle, add the ſquare of the central diſtance; from the ſum, take a third of the ſquare of the length; multiply the remainder by the length; call the product B.

Or $B = \overline{\overline{rr + xa} - \frac{1}{3}\,ll} \times l$.

Then A taken from B, and the remainder multiplied by 3,1416 will give the ſolidity required.

Ex.

EXAMPLE.

What is the ſolidity of a fruſtum of a circular ſpindle; the diameter of the end paſſing thro' the centre being 36; that of the leſſer end 16; and their diſtance 20?

Now 7 is the central diſtance, and 25 is the radius.

And 879,5635 is the area of the zone.

Therefore (879,5635×7=) 6156,9445=A.

Alſo ($\overline{\overline{25}|^2 + \overline{7}|^2 - \frac{1}{3} \times \overline{20}|^2} \times 20 =$) 10813,3 = B.

Therefore B — A = 4656,3888.

Then (4656,3888 × 3,1416 =) 14628,511 is the ſolidity ſought.

This rule is of uſe in finding the content of ſuch caſks, as are in the form of circular ſpindles.

PRO-

PROPOSITION XXVI.

To find the ſuperficial content of a circular ſpindle, whoſe length and greateſt diameter are known.

RULE.

Find the radius of the prime circle, and the central diſtance; and alſo, the length of the arc of the generating ſegment.

Multiply the radius of the prime circle by the length of the ſpindle; divide the product by the central diſtance; from the quotient ſubtract the length of the arc; multiply the remainder by twice the central diſtance; the product multiplied by 3,1416, will give the ſuperficies required.

EXAMPLE.

What is the ſuperficial content of a circular ſpindle whoſe length is 48; and its greateſt diameter is 36?

Now 25 will be found for the radius; 7 for the central diſtance; and 64,34496 for the length of the arc.

Then $\left(\overline{\frac{25\times48}{7}-64{,}34496}\times7\times2\times3{,}1416=\right)$ 4709,794 is the ſuperficies required.

PRO-

PROPOSITION XXVII.

To find the convex ſuperficies of the fruſtum of a circular ſpindle, wherein one end paſſes thro' the centre: The diameters of thoſe ends, and their diſtance being known.

RULE.

Find the central diſtance, and the radius of the prime circle.

Seek the degrees (in tables) correſponding to a ſine found by dividing the given length of the fruſtum, by the radius of the prime circle; multiply theſe degrees by 0,0174534, and by the central diſtance; ſubtract the product from the length of the fruſtum; the remainder multiplied by the circumference of the prime circle, will give the convex ſurface required.

EXAMPLE.

What is the convex ſurface of the fruſtum of a circular ſpindle; the diameter of the end paſſing thro' the centre, being 36; that of the leſſer end 16; and their diſtance 20?

Now 7 is the central diſtance, and 25 is the radius of the prime circle.

And $\frac{20}{25}$ = equal 0,8 is the ſine of 53,13 degrees.

Alſo (25×2×3,1416=)157,08 is the circumfe.

Then 53,13×0,0174534×7=6,49103.

Therefore (20−6,49103×157,08=) 2121,989 is the ſuperficies ſought.

SEC-

SECTION V.

PRACTICAL QUESTIONS.

QUESTION I.

HOW many 3 inch cubes may be cut out of a 12 inch cube?

Now $\frac{12}{3} = 4$ the number of 3 inches in one ſide of a 12 inch cube.

Then $4 \times 4 \times 4 = 64$, the number of 3 inch cubes contain'd in a 12 inch cube.

QUESTION II.

A farmer borrow'd of his neighbour a piece of a hayrick, which meaſured 6 feet every way; (that is, a cube whoſe ſide was 6 feet,) and the borrowing farmer, paid back two equal cubical pieces, each of whoſe ſides were three feet; Query whether the lending farmer was fully paid?

Now $6 \times 6 \times 6 = 126$, the ſolidity of the piece borrowed.

And $3 \times 3 \times 3 = 27$.

Therefore $27 \times 2 = 54$, the ſolidity of the pieces paid.

Then $\frac{216}{54} = 4$; therefore the lending farmer was paid but a fourth.

QUES-

QUESTION III.

One beſpoke an iron roller for a garden, the outſide diameter was to be 20 inches; length of the roller, 50 inches; and thickneſs of the metal, $1\frac{1}{2}$ inch; now ſuppoſing every cube inch weighs $4\frac{1}{4}$ ounces; what will the whole come to at $3\frac{1}{4}$ d. ⅌ ℔?

Now $1\frac{1}{2} \times 2 = 3$, the double thickneſs; and $20 - 3 = 17$, the inner diameter.

Then $20 \times 20 \times 0{,}7854 = 314{,}16$; and $17 \times 17 \times 0{,}7854 = 226{,}9806$.

Therefore $314{,}16 - 226{,}9806 = 87{,}1794$.

And $87{,}1794 \times 50 = 4358{,}97$ the ſolidity of the roller.

But $4\frac{1}{4}$ *oz.* $= 0{,}265625$ ℔.

Therefore 1 *inch.* : 0,265625 ℔ : : 4358, 97 *inch.* : 1157,8514, *&c.* the weight.

And $3\frac{1}{4}$ *d.* $= 0{,}0135416$ £.

Therefore 1 ℔ : 0,0135416 £. : : 1157,8514 : 15,679 £. *&c.* = 15 £. 13 *s.* 7 *d.* the whole value of the roller.

QUES-

QUESTION IV.

A Maſon has ſet up two ſtone lamp-poſts, each conſiſting of a 16 inch ſquare pyramidal ſhaſt of 7 feet high; and a parallelopiped pedeſtal of 18 inches ſquare, and 3 feet high: What will they come to, at 2 s. 6 d. a foot.

Now 18 inches $=$ 1,5 feet, one ſide of the baſe of the pedeſtal.

And $1,5 \times 1,5 \times 3 = 6,75$ feet, the ſolidity of one pedeſtal.

Alſo 16 inches $= 1,\dot{3}$ feet.

Now $1,\dot{3} \times 1,\dot{3} \times \frac{7}{3} = 4,14\dot{8}$, the ſolidity of one pyramid.

Then $6,75 + 4,14\dot{8} = 10,89\dot{8}1\dot{4}$ the ſolidity of one poſt.

And $10,89814 \times 2 = 21,7963$ the ſolidity of both.

Now 2 *s.* 6 *d.* $=$ 0,125 £.

Then 1 f. : 0,125 £. : : 21,7963 : 2,724 £.

Or 2 £. 14 *s.* 6 *d.* the whole expence.

QUES-

QUESTION. V.

A, has a cone of marble, the circumference of whoſe baſe is 37,6992 *inches, and the height* 24; *which he would exchange with B, for a fine porphyry octagonal pyramid of the ſame height; and each ſide of the baſe is* 5 *inches: B inſiſts to be paid* 3 *s. an inch for the difference of the ſolidities: What ſum will he receive?*

Now $37{,}6992 \times 37{,}6992 \times 0{,}026529 \times 24 =$ $904{,}7885$ the ſolidity of the cone.

And $5 \times 5 \times 4{,}828427 \times \frac{24}{3} = 965{,}6854$ the ſolidity of the pyramid.

Therefore $965{,}6854 - 904{,}7885 = 60{,}8969$ the difference of the ſolidities.

Then if 1 inch : 0,15 £. : : 60,8969 : 9,1345 £.

Or 9 £. 2 *s.* $8\frac{1}{2}$ *d.* the ſum *A* muſt give to boot.

QUESTION VI.

What will the painting a conical church ſpire come to at 8 *d.* ⅌ *yard; ſuppoſing the circumference of the baſe* 64 *feet, and the altitude* 118 *feet?*

Now 1 : 0,318309 : : 64 : 20,371776, the diameter of the baſe.

And $\frac{20{,}371776}{2} = 10{,}185888$, the radius.

Again, $118 \times 118 = 13924$; and $10{,}185888 \times 10{,}185888 = 103{,}7523$, *&c.*

Then $13924 + 103{,}7523 = 14027{,}7523$; whoſe ſquare root is 118,43 feet, the ſlant ſide.

Therefore $118,43 \times \frac{64}{2} = 3789,76$ feet; and

$\frac{3789,76}{9} = 421,084$ yards.

Now 1 *yd.* : 0,03 £. : : 421,084 *yds.* : 14,036140 £. = 14 £. 0 *s.* 8¼ *d.* and ſo much will be the whole expence.

QUESTION VII.

One has in his garden, a regular octagonal pyramid, of 7 feet high, with a cubical dial fixed to its vertex; one ſide of the cube is equal to one ſide of the pyramid's baſe, which is 9 inches; what will the gilding this pyramid and cube come to, at 2 d. a ſquare inch?

To ſolve this queſtion.

1ſt, Find the length of a line drawn from the centre of the baſe, to the middle of one of its ſides.

2dly, Find the length of a line drawn from the vertex of the pyramid, to the middle of one ſide in the baſe, this line will be the perpendicular height of one of the 8 triangles. The reſt is obvious.

Now $9 \times 1,2071068$ (tab. I. p. 144.) $= 10,863969$ is the radius of the inſcrib'd circle; or equal to the diſtance of the centre of the baſe from the middle of one ſide.

And 7 feet = 84 inches.

Then $\sqrt{\overline{10,863965}|^2 + \overline{84}|^2} = 84,7$ is the ſlant ſide of the pyramid, or height of one of the triangles compoſing the pyramid.

Then $84,7 \times \frac{9}{2} \times 8 = 3049,2$ the pyramid's ſuperficies.

Alſo $9 \times 9 \times 6 = 486$ the ſuperficies of the cube.

Then $3049,2 + 486 = 3535,2$ the ſuperficies of the pyramid and cube.

And 1 inch. : 0,0083 £. (= 2 *d.*) : : 3535,2 : 29,46 £.

Or 29 £. 9 *s.* 2½ *d.* the whole expence.

QUESTION VIII.

What will a marble fruſtum of a cone come to, at 12 *s. a foot ſolid; the diameter of the greater end being* 4 *feet; that of the leſſer end,* 1½ *feet; and the length of the ſlant ſide* 8 *feet.*

Now $\frac{4 - 1,5}{2} = 1,25$.

And $1,25 \times 1,25 = 1,5625$.

Alſo $8 \times 8 = 64$.

Then $64 - 1,5625 = 62,4375$, whoſe ſquare root is 7,9 feet, the altitude of the fruſtum.

Alſo $4 \times 4 = 16$; $1,5 \times 1,5 = 2,25$; $4 \times 1,5 = 6$.

And $16 + 2,25 + 6 = 24,25$.

Then $24,25 \times 7,9 \times 0,2618 = 50,154335$ feet, the ſolidity of the fruſtum.

There. 1 *f.* : 0,6 £. : : 50,154335 f. : 30,092601 £. = 30 £. 1 *s.* 10 ¼ *d.* the expence of the whole.

QUES-

QUESTION IX.

Suppoſe a church-ſpire was to be built of an octagonal form; one ſide of the greater end to be 24 feet; one ſide of the leſſer end, to be 12,5 feet; and the height 80 feet; but the inſide of the ſpire is to be run up in a conical form; the diameter of the baſe is to be 56 feet; and the diameter at top to be 28 feet; what will be the expence, at 4 s. 6 d. ℘ foot ſolid?

Now $12{,}5 \times 24 = 300$, the product of the ſides of the ends.

And $24 - 12{,}5 = 11{,}5$ and $11{,}5 \times 11{,}5 = 132{,}25$.

Conſequently $\frac{132{,}25}{3} = 44{,}08\dot{3}$ = to $\frac{1}{3}$ of the ſquare of the difference of the two ſides.

Then $\overline{300 + 44{,}08\dot{3}} \times 80 \times 4{,}828427 = 132910{,}5$ the ſolidity of the fruſtum of the pyramid.

Again $56 \times 28 = 1568$; $56 - 28 = 28$; $28 \times 28 = 784$; $\frac{784}{3} = 261{,}\dot{3}$; and $1568 + 261{,}\dot{3} = 1829{,}\dot{3}$.

Then $1829{,}\dot{3} \times 80 \times 0{,}7854 = 114940{,}672$, the ſolidity of the contain'd cone.

Therefore $132910{,}5 - 114940{,}672 = 17969{,}828$ feet, the ſolidity of the ſtone work.

Then 1 f. : 0,225£. :: 17969,828 f. : 4043,2113£. = 4043£. 4s. 2¾d. the coſt.

QUES-

QUESTION X.

A maſon is employ'd to compleat a decayed portland ſtone cone; he having made the upper part level, the meaſures of the fruſtum are as follows; length of the ſlant ſide 12 feet; diameter of the upper end 6 feet; and the circumference of the baſe 38 feet; what will it coſt at 3s. ⅌ foot, to put a piece on, equal to that taken off?

Now 1 inch : 0,318309 : : 38 : 12,095742, or 12,1, the diameter of the baſe.

And 12,1 — 6 = 6,1.

Therefore $\frac{6,1}{2}$ = 3,05, the half difference of the diameters.

Then 12 × 12 = 144; 3,05 × 3,05 = 9,3025; and 144 — 9,3025 = 134,6975, whoſe ſquare root is 11,6, &c. the fruſtum's height.

Then 6,1 : 11,6 : : 6 : 11,4, &c. the height of the piece wanting.

And $6 \times 6 \times 0,7854 \times \frac{11,4}{3} = 107,44272$, &c. feet, the ſolidity of the piece wanting.

Then 1 f. : 0,15£. : : 107,44272 f. : 16,1164, = 16£. 2*s.* 4*d.* the whole expence to compleat the cone.

QUESTION XI.

Three men bought a tapering piece of timber, which was the fruſtum of a ſquare pyramid; one ſide of the greater end was three feet; one ſide of the leſſer end, 1 foot; and the length, 18 feet; and they paying equally, are to have equal ſhares; what is the length of each man's piece?

Now $\left(\overline{3\times1+\frac{3-1\times3-1}{3}}\times18=\right)$ 78 the ſolidity of the fruſtum, by Prop. VII.)

And $\left(\frac{78}{3}=\right)$ 26, the ſolidity of each man's ſhare.

Alſo $\left(\frac{18\times3}{3-1}=\right)$ 27 is the length of the pyramid. (by Prop. X.)

And $27 - 18 = 9$ is the length of the piece wanting.

Whoſe ſolidity is $\left(1\times1\times\frac{9}{3}=\right)$ 3.

Therefore $78+3=81$ is the ſolidity of the pyramid.

Now $81 : \overline{27}|^3 :: 26+3 : 7047$; whoſe cube root is 19,172.

Alſo $81 : \overline{27}|^3 :: 3+26\times2 : 13365$; whoſe cube root is 23,731.

Then $27 - 23{,}731 = 3{,}269$ for the length of the 1ſt man's ſhare.

And $23{,}731 - 19{,}172 = 4{,}559$ the length of the 2d man's ſhare.

And $19{,}172 - 9 = 10{,}172$ the length of the 3d man's ſhare, reckoning the ſhares from the greater end towards the leſſer.

QUESTION XII.

Two men purchaſe a piece of Egyptian granite, which is a conical fruſtum, with parallel ends; whoſe greater diameter is 50 *inches; the leſſer* 20; *and the length of the ſlant ſide,* 40 *inches: And they agree to cut it, parallel to the ends, into two pieces of equal ſolidity; and he who takes the piece next to the greater end, ſhall pay to the other,* 2 *d. for every ſquare inch difference between the whole ſuperficies of the two pieces: How much will he receive, who takes the leſſer end?*

Now $\sqrt{\overline{40}|^2 - \overline{\frac{50-20}{2}}\Big|^2} = 37$ inches, is the height of the fruſtum.

And $30 : 37 :: 20 : 24{,}6$, the height of the piece wanting.

Th. $37 + 24{,}6 = 61{,}6$, the cone's height.

Alſo $\overline{50}|^2 \times \frac{61{,}6}{3} \times 0{,}7854 = 40360{,}83$ the cone's ſolidity.

And $\overline{20}|^2 \times \frac{24{,}6}{3} \times 0{,}7854 = 2583{,}093$, the ſolidity of the piece wanting.

Therefore $40360{,}83 - 2583{,}093 = 37777{,}74$, the ſolidity of the given fruſtum.

And $\frac{37777{,}74}{2} = 18888{,}87$ is the ſolidity of each man's ſhare.

Alſo $40360{,}83 : \overline{61{,}6}|^3 :: 2583{,}093 + 18888{,}87 : 124756{,}454$ whoſe cube root is 49,9, &c. or 50 inches.

Then ($50 - 24,\acute{6} =$) $25,\acute{3}$ inches meaſured from the leſſer end, and parallel to the axis, will give the place in the fruſtum where the ſection is to be made.

And $61,\acute{6} : 50 :: 50 : 40,54$, the diameter at the ſection.

Now $37 : 40 :: 25,\acute{3} : 27,387$ the ſlant ſide of the upper fruſtum.

And $40 - 27,387 = 12,613$ the ſlant ſide of the lower fruſtum.

Alſo $\overline{50+40,54} \times 1,5708 \times 12,613 = 1793,8236$ the convex ſurface of the lower fruſtum.

And $\overline{20+40,54} \times 1,5708 \times 27,387 = 2604,4103$ the convex ſurface of the upper fruſtum.

Likewiſe $\overline{50}|^2 \times 0,7854 = 1963,5 =$ area at the greater end.

And $\overline{20}|^2 \times 0,7854 = 314,16 =$ area at the leſſer end.

Alſo $\overline{40,54}|^2 \times 0,7854 = 1290,7981 =$ area at the ſection.

Then $1793,8236 + 1963,5 + 1290,7981 = 5048,1217$ the ſuperficies of the lower fruſtum.

And $2604,4103 + 1290,7981 + 314,16 = 4209,3688$ the ſuperficies of the upper fruſtum.

Therefore $5048,1217 - 4209,3688 = 838,7529$ inches the difference of the ſuperficies. Which, at 2*d.* an inch, amounts to 6,989£.; or 6£. 19*s.* 9½*d.* the money he is to receive, who takes the fruſtum next the leſſer end.

QUESTION XIII.

Suppoſe the globe, or ball, on the top of St. Paul's *church to be* 6 *f. in diameter; what did the gilding thereof come to at* $3\frac{1}{2}$ *d.* ⅌ *inch ſquare?*

Now 6 feet equal 72 inches.

And $72 \times 72 \times 3{,}1416 = 16286{,}0544$ the ſuperficies.

Then 1 inch : ($3\frac{1}{2}$ *d.* =) 0,014583 £. : : 16286,0544 : 237,9935 £.; or 237 £. 19 *s.* $10\frac{1}{2}$ *d.* the expence.

QUESTION XIV.

What is the weight of a bomb-ſhell, whoſe outſide diameter is 16 *inches; and the thickneſs of the metal,* 3 *inches; ſuppoſing a cubic inch weighs* $4\frac{1}{2}$ *ounces?*

Now $16 - \overline{3 \times 2} = 10$ inches, the inſide diameter.

And $16 \times 16 \times 16 \times 0{,}5236 = 2144{,}6656$.

Alſo $10 \times 10 \times 10 \times 0{,}5236 = 523{,}6$ the cubic inches in the concavity.

Therefore $2144{,}6656 - 523{,}6 = 1621{,}0656$ the ſolidity the ſhell.

But $4\frac{1}{2}$ oz. = 0,28125 ℔.

Then 1 : 0,28125 : : 1621,0656 : 455,9247 ℔ weight.

QUESTION XV.

A perſon wants a cylindric veſſel of 3 feet deep, that ſhall hold twice as much as a veſſel of 28 inches deep, and 46 inches in diameter throughout. What muſt be the diameter of the required veſſel?

Now $\overline{46}|^2 \times 0,7854 \times 28 = 46533,5792$, the tents of the given veſſel.

And $46533,3792 \times 2 = 93066,7584$, the contents of the required veſſel.

Alſo $\frac{93066,7584}{36\ (=3\text{f.})} = 2585,1877$, the area of the required veſſel's baſe.

Then $1 : 1,2732 :: 2585,1877 : 3291,8$, whoſe ſquare root is 57,37 the diameter of the required veſſel's baſe.

QUESTION XVI.

One has a Grainery 47 feet 8 inches long, 18 feet 5 inches broad, and 9 feet 7 inches high; but wants another that will hold four times as much, and have the dimenſions in the ſame proportion to each other as the old one has; what will be the length, breadth and depth of the new one?

Now 47 feet 8 inches = $47,\dot{6}$; 18 feet 5 inches = $18,41\dot{6}$; 9 feet 7 inches = $9,58\dot{3}$.

Then $18,41\dot{6} \times 9,58\dot{3} \times 47,\dot{6} = 8412,836$, the ſolid content of the old Grainery.

And $8412,836 \times 4 = 3361,344$, the ſolid content of the new one.

Alſo $\overline{47,\dot{6}}^3 = 108303,9\dot{6}$, the cube of the length of the old one.

Then $8412,836 : 108303,9\dot{6} :: 33651,344 : 433215,866$, *&c.* whoſe cube root is 75,666, *&c.* the length of the new one.

And $47,\dot{6} : 75,\dot{6} :: 18,41\dot{6} : 29,23$, *&c.* the breadth of the new one.

Therefore $47,\dot{6} : 75,6 :: 9,58\dot{3} : 15,21$ nearly, the depth or height of the new one.

QUESTION XVII.

A gentleman is deſirous of having in his park, a rectangular canal, of a quarter of a mile long, that ſhall contain 4 *acres on the ſurface; be* 7 *feet deep; and the ſides and ends to ſlope in the diagonal of a ſquare whoſe ſide ſhall be the depth of the canal: What will the digging come to at* 4 *s.* 6 *d. a floor* (*of* 18 *feet ſquare, and* 1 *foot deep;*) *for work and carriage?*

An acre = 4840 ſquare yards.

And 4 acres = 19360 ſquare yards.

A quarter of a mile, is 440 yards.

Or 1320 feet, the length of the canal at the upper ſurface.

Then $\frac{19360}{440} = 44$ yards.

Or 132 feet for the breadth of the canal at the upper ſurface.

And $1320 - \overline{7 \times 2} = 1306$, the length at bottom.

And $132 - \overline{7 \times 2} = 118$, the breadth at bottom.

Then by Prop. 12.

$$\overline{132 + \frac{118}{2}} \times 1320 = 252120.$$

And $\overline{118 + \frac{132}{2}} \times 1306 = 240304.$

Then $\overline{252120} + \overline{240304} \times \frac{7}{3} = 1148989,3$ feet the content of the canal.

And $\frac{1148989,3}{324\,(=18 \times 18)} = 3546,26$ floors.

Then 1 floor : 0,225 £. (= 4 *s.* 6 *d.*) : : 35462,62 : 797,0985.

Or 797 £. 18 *s.* 2 *d.* the expence.

QUESTION XVIII.

One who had been to gauge a veſſel, which was the fruſtum of a cone, with parallel end, and 9 *feet long; found it held* 404,2638 *gallons of beer meaſure: And being aſked for the diameters, ſaid, he had forgot them, but that one was twice as much as the other: What were thoſe diameters?*

Note, A gallon, beer meaſure, is equal to 282 cubic inches.

Then $\frac{404,2638 \times 282}{1728} = 65,9736$ the ſolidity of the veſſel in feet.

Now $\frac{65,9736}{9} = 7,3304$, the mean area, between the areas of the ends of the veſſel.

And (as the area of a circle) 1 : (to the ſquare of the diameter) 1,2732 £. : : (ſo is any other area =) 7,3304 : (to the ſquare of the diameter) 9,ʒ.

Between the ſquares of the terms 2 and 1, of the given ratio of the diameters, find a mean ſquare.

Thus $\overline{2 \times 1} + \frac{\overline{2-1} \times \overline{2-1}|}{3} = 2,ʒ$ the mean ſquare.

Then (as any mean area =) 2,ʒ : (to the ſquare of the greater extream =) 4 : : (ſo is any other mean area =) 9,ʒ : (to the ſquare of its greater extream =) 16; whoſe ſquare root 4, is the greater diameter.

And : 2 1 : : 4 : 2 feet the leſſer diameter.

QUES-

QUESTION XIX.

Suppoſe two porters having a quart of ſtrong beer between them, agree to drink it off at two pulls, that is, a draught to each; now the firſt having given it the black eye, as they call it, that is, drank till the ſurface of the liquor touch'd the oppoſite edge of the bottom, he gave the remaining part of it to the other; what was the difference of their ſhares? Suppoſing the quart pot was the fruſtum of a cone; the depth being 5,7 inches, the diameter at top 3,7 inches, and the ſolidity 70,5 ſolid inches?

Firſt, The diameter of the bottom muſt be found, and then the difference of the ſolidities of the two hoofs.

Having given the ſolidity, length, and one of the diameters of a cone's fruſtum, the other diameter may be found by the following

RULE.

Multiply the length by 0,2618, and divide the given ſolidity by this product: From the quotient ſubtract $\frac{3}{4}$ of the ſquare of the given diameter; from the ſquare root of the remainder, take half the given diameter, and it will leave the diameter ſought.

Now $0,2618 \times 5,7 = 1,49226$.

And $\frac{70,5}{1,49226} = 47,243$.

Alſo $3,7 \times 3,7 \times \frac{3}{4} = 10,2675$.

And $47,243 - 10,2674 = 36,9755$ whoſe ſquare root is, 6,08, *&c.*

Then $6,08 - \frac{3,7}{2} = 4,23$ the diameter of the bottom of the pot.

Now $4,23 \times 4,23 \times 4,23 = 75,686967$.

And $3,7 \times 3,7 \times 3,7 = 50,653$.

Alſo $75,686967 \times 50,653 = 3833,771939$, whoſe ſquare root is 61,917.

Then $61,917 - 50,653 = 11,264$.

And $4,23 - 3,7 = 0,53$.

Alſo $\frac{11,264}{0,53} = 21,253$.

Then $21,253 \times 5,7 \times 0,2618 = 31,715$ cubic inches, what the firſt man drank.

And $70,5 - 31,715 = 38,785$ cubic inches, what the laſt man drank.

Then $38,785 - 31,715 = 7,07$ cubic inches, the difference of their ſhares.

PART

Fig. 1.

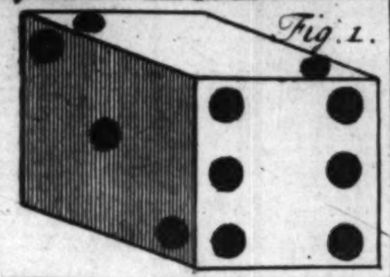


Fig 2

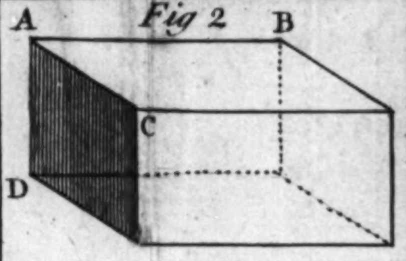
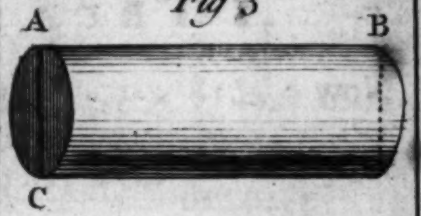


Fig 3



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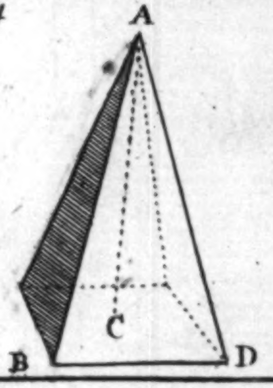


Fig 5

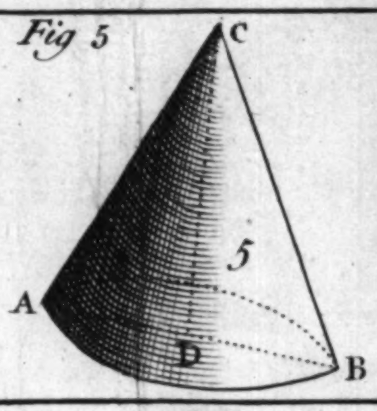


Fig 6

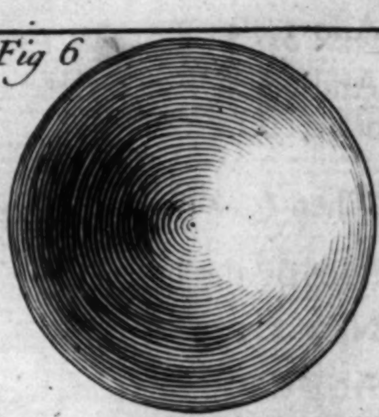


Fig 7

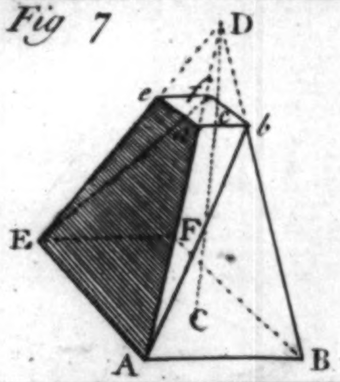


Fig 8

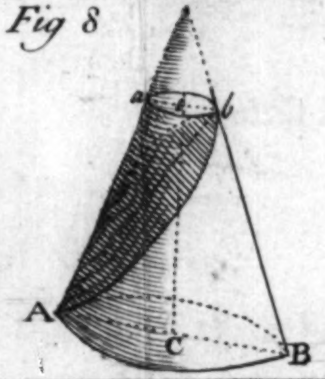


Fig 9

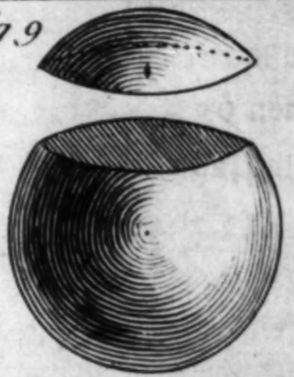


Fig 10

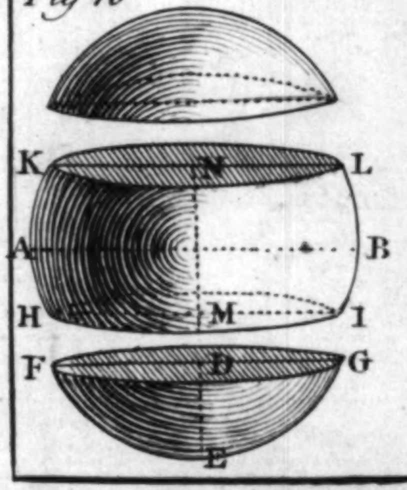
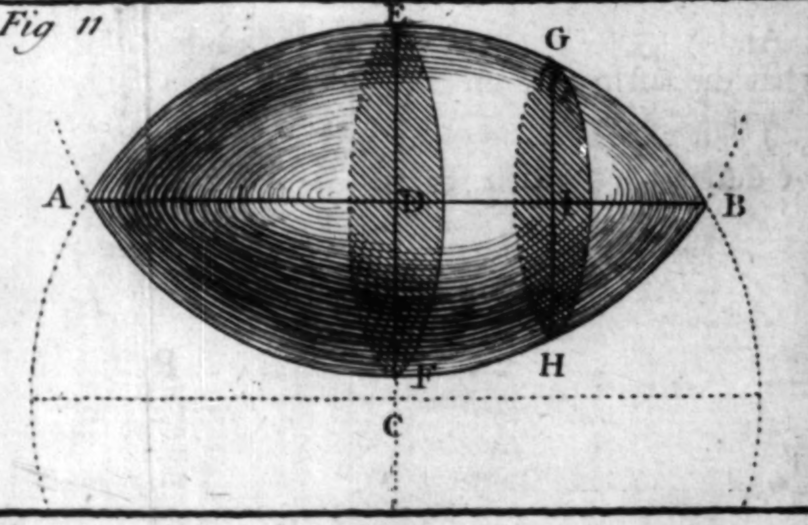


Fig 11



PART III.

SECTION I.

I. IF a right cone (ABC) be cut by a plane (DE) parallel to its baſe, (BC), the ſection will be a *circle*. (Plate III. Fig. 1.)

II. If the plane (DF) paſs thro' the oppoſite ſides (AC, AB) of the cone, the ſection will be an *ellipſis*. (Fig. 1, 2.

III. If the plane (DG) paſs parallel to one ſide (AB) of the cone, thro' the oppoſite ſide (AC), and baſe (BC), the ſection will be a *parabola*. (Fig. 1, 3.)

IV. If the plane (DH) paſs thro' one ſide (AC) of the cone, and the baſe (BC), in ſuch a manner, as being continued upwards (in DI), would meet the oppoſite ſide (BA), of the cone continued upwards, (in AI), the ſection will be a *Hyperbola*. (Fig. 1, 4.)

V. The point (D) of the curve, neareſt the vertex (A) of the cone (ABC), and where the curve is moſt acute, is called *the principal vertex*. (Fig. 1.)

VI. A right line (AB) drawn thro' the principal vertex (A), dividing the area of the ſection into two equal parts, is called the *axis*. (Fig. 2, 3, 4.)

VI. In

VII. In the ellipſis, this line (AB) is called the *tranſverſe diameter*, or *tranſverſe axe*. (Fig. 2.)

VIII. In the hyperbola, the continuation (DI), of the axe, till it meets the other ſide (BA, of the cone, continued, is called the *tranſverſe diameter*. (Fig. 1.)

IX. A right line (PH=2PI), drawn at right angles to the axe (AB), and terminated at each end by the curve, is called a *double ordinate*. (Fig. 2, 3, 4.)

X. The double ordinate (DK) paſſing thro' the middle (C) of the tranſverſe diameter (AB) in the ellipſis, is called the *conjugate diameter*, or *conjugate axe*. (Fig. 2.)

XI. That part (AI, or GD), of the axe (AB, or DK) intercepted between the curve and the ordinate (PI or GP), is called an *abſciſs*. (Fig. 2, 3, 4.)

XII. A curved priſmatic figure, with ſtrait ſides, and parallel equal elliptic ends, is called a *cylindroid*.

XIII. A figure, formed by the rotation of an ellipſis, round either of its axes, is called a *ſpheroid*: The fixed axis is called the *axe of rotation*; and the other axis is called the *revolving axe*.

XIV. If the tranſverſe be the axe of rotation, the figure generated is called a *prolate ſpheroid*.

XV. If the conjugate be the axe of rotation, the figure generated is called an *oblate ſpheroid*.

XVI. A figure generated by the rotation of a parabola, or a hyperbola, about its axe, is called a *conoid.*

XVII. If by a parabola, 'tis called a *parabolic conoid*, or *paraboloid.*

XVIII. If by a hyperbola, 'tis called a *hyperbolic conoid*, or *hyperboloid.*

Note, If a plane cut a ſpheroid, or conoid, oblique to the axe, the ſection will be elliptical.

XIX. A figure ſuppoſed to be generated by the rotation of a ſegment of an ellipſis, or of a parabola, about its ordinate, is called a *ſpindle*, and is denominated, either as *elliptic*, *parabolic*, or *hyperbolic.*

XX. A part of either of theſe ſolids, contained between two parallel ſections, or plane ends, is called a *fruſtum:* And a part with only one plane end, is called a *ſegment.* (Fig. 7, 8, 12.)

SECTION II.

Of elliptical lines, ſuperficies, and ſolids.

PROPOSITION I.

In an ellipſis, any three of theſe four terms being given; viz. *half tranſverſe*, (CB=t) *half conjugate*, (CD=c) *ordinate*, (FE=y) *abſciſſa*, (CE=x) *the other is eaſily found.* (Plate III. Fig. 2.)

CASE I. *When the half tranſverſe, half conjugate, and abſciſs are known, to find the ordinate.*

RULE.

From the ſquare of the half tranſverſe, take the ſquare of the abſciſs; multiply the ſquare root of the remainder by the quotient of the half conjugate, divided by the half tranſverſe, and the product is the ordinate.

Or $y = \frac{c}{t}\sqrt{tt-xx}$.

EXAMPLE.

Suppoſe the half tranſverſe (CB) *is* 60; *the half conjugate* (CD) *is* 20; *and the abſciſs* (CE) *is* 36: *Required the ordinate* EF?

Then $\left(\frac{20}{60}\times\sqrt{60^2-36^2}=\right)$ 16 is the ordinte.

II. *When the half tranſverſe, half conjugate, and ordinate are known; to find the Abſciſs.*

RULE.

From the ſquare of the half conjugate, take the ſquare of the ordinate; multiply the ſquare root of the remainder, by the quotient of the half tranſverſe, divided by the half conjugate, and the product is the abſciſs.

Or $x = \frac{t}{c}\sqrt{cc - yy}$.

EXAMPLE.

Suppoſe the half tranſverſe (CB) *is* 60; *the half conjugate* (CD) *is* 20; *and the ordinate* (FE) *is* 16: *Required the abſciſs* (CE)?

Then $\left(\frac{60}{20} \times \sqrt{20 \times 20 - 16 \times 16} =\right) 36$ is the abſciſs.

III. *The half conjugate, the ordinate, and abſciſs being known; to find the half tranſverſe.*

RULE.

From the ſquare of the half conjugate, take the ſquare of the ordinate; make the ſquare root of the remainder, a diviſor to the product of the half conjugate by the abſciſs; and the quotient will be the half tranſverſe.

Or $t = \frac{cx}{\sqrt{cc-yy}}$

EXAMPLE.

Suppoſe the half conjugate (CD) *is* 20; *the ordinate* (EF) *is* 16; *and the abſciſs* (CE) *is* 36: *Required the half tranſverſe* (CB)?

Then $\left(\frac{20\times 36}{\sqrt{20\times 20 - 16\times 16}} =\right)$ 60 is the half tranſverſe.

IV. *The half tranſverſe, the ordinate, and the abſciſs being known; to find the half conjugate.*

RULE.

From the ſquare of the half tranſverſe, take the ſquare of the abſciſs; let the ſquare root of the remainder, be a diviſor to the product of the half tranſverſe by the ordinate; and the quotient will be the half conjugate.

Or $c = \frac{ty}{\sqrt{tt - xx}}$

EXAMPLE.

Suppoſe the half tranſverſe (CB) *is* 60; *the ordinate* (EF) *is* 16; *and the abſciſs* (CE) *is* 36: *Required the half conjugate?*

Then $(\frac{60 \times 16}{\sqrt{60 \times 60 - 36 \times 36}} = 20$ is the half conjugate.

PRO-

PROPOSITION II.

To find the periphery of an ellipſis, the tranſverſe and conjugate diameters, or axes, being known.

RULE.

Multiply half the ſum of the two diameters, by 3,1416; and the product will be the periphery, exact enough for moſt practical purpoſes.

EXAMPLE.

What is the periphery of an ellipſis, whoſe tranſverſe axe is 24, *and the conjugate* 18.

Then $\left(\frac{24+18}{2} \times 3,1416 =\right) 65,9736$ is the periphery ſought.

The periphery of an ellipſe may be found nearly accurate to five places, by the following table, conſtructed by Sir *Jonas Moore*; who ſays of it, "I have "made above 45000 arithmetical operations for this "table, and am now well pleaſed it is finiſhed. Some "perhaps may find ſhorter ways, as I believed I had "myſelf, till adviſed otherwiſe by the truly honour"able the lord *Bruncker*. I therefore purſued the "rules given by me, in contemplation of the ellipſis "printed in my arithmetic, taking 100 ellipſes be"twixt that which falls upon the diameter (equal in "this caſe to 2,0000; the firſt in the table), and the "greateſt which is the circle, (the laſt)."

A TABLE for finding the Periphery of an Ellipſis.

A.	Periph.	D.	A.	Periph.	D.	A.	Periph.	D.
1	2,0012	12	34	2,2368	104	67	2,6465	139
2	2,0028	16	35	2,2474	106	68	2,6604	140
3	2,0048	20	36	2,2581	107	69	2,6744	140
4	2,0072	26	37	2,2692	111	70	2,6884	141
5	2,0100	28	38	2,2803	111	71	2,7025	141
6	2,0133	33	39	2,2915	112	72	2,7166	143
7	2,0170	37	40	2,3028	113	73	2,7309	144
8	2,0213	43	41	2,3142	114	74	2,7453	146
9	2,0261	48	42	2,3256	114	75	2,7599	146
10	2,0314	53	43	2,3371	115	76	2,7745	146
11	2,0370	56	44	2,3488	117	77	2,7891	147
12	2,0432	62	45	2,3607	119	78	2,8038	148
13	2,0496	64	46	2,3726	119	79	2,8186	148
14	2,0564	68	47	2,3848	122	80	2,8334	148
15	2,0634	70	48	2,3970	122	81	2,8482	148
16	2,0708	74	49	2,4094	124	82	2,8630	149
17	2,0784	76	50	2,4218	124	83	2,8779	150
18	2,0862	78	51	2,4342	124	84	2,8929	150
19	2,0942	80	52	2,4467	125	85	2,9080	151
20	2,1024	82	53	2,4594	127	86	2,9231	151
21	2,1106	82	54	2,4723	129	87	2,9382	152
22	2,1192	86	55	2,4852	129	88	2,9534	152
23	2,1281	89	56	2,4983	131	89	2,9686	153
24	2,1373	92	57	2,5114	131	90	2,9839	154
25	2,1467	94	58	2,5245	131	91	2,9993	154
26	2,1561	94	59	2,5377	132	92	3,0147	155
27	2,1658	97	60	2,5510	133	93	3,0302	156
28	2,1756	98	61	2,5644	134	94	3,0458	156
29	2,1856	100	62	2,5779	135	95	3,0614	157
30	2,1956	100	63	2,5915	136	96	3,0771	157
31	2,2057	101	64	2,6052	137	97	3,0928	158
32	2,2160	103	65	2,6189	137	98	3,1086	158
33	2,2264	104	66	2,6327	138	99	3,1244	158

The uſe of the table, for finding the periphery of an ellipſe, whoſe tranſverſe and conjugate axes are known.

RULE.

Divide the conjugate by the tranſverſe; if the quotient gives no more than two places, ſeek them in the columns ſigned *axe*; the number right againſt it, in the column ſigned *periphery*, multiplied by the given tranſverſe, will give the periphery required.

EXAMPLE I.

Let the tranſverſe be 1; *and the conjugate* 0, 5: *Required the periphery?*

Now $\frac{05}{1} = 0{,}50$.

And againſt 50 (found in the column ſigned *axe*) is 2,4218; (found in the column ſigned *periphery*). Then (2,4218 × 1 =) 2,4218 is the periphery ſought.

EXAMPLE II.

Let the tranſverſe be 24, *and the conjugate* 18: *Required the periphery?*

Now $\frac{18}{24} = 0{,}75$: Againſt it is 2,7599.

Then (2,7599 × 24 =) 66,2376 is the periphery required.

If the quotient of the conjugate by the tranſverſe exceed two places.

RULE.

1ſt. Seek the two firſt places of the quotient in the column ſigned *axe*, and take out the periphery againſt it; and alſo the number next below it, in the column ſigned *diff.*

2d. Multiply this diff. by the remaining part of the quotient; write the two left hand places under the two right hand places of the ſaid periphery; their ſum multiplied by the given tranſverſe, will give the periphery required.

EXAMPLE III.

The longer axe 10000, *and the ſhorter axe* 4382: *Required the periphery?*

Now $\frac{4382}{10000} = 0,4382$.

Againſt 43, and under periph. is 2,3371.
Between 43 and 44, and under diff. is 117.
And $117 \times 82 = 9594$ or 96, or 0,0096.
Then $\overline{2,3371 + 0,0096} \times 10000 = 23467$ for the periphery required.

EXAMPLE IV.

Suppoſe the tranſverſe diameter is 32,54, *and the conjugate* 18,64; *Required the periphery?*

Now $\frac{18,64}{32,54} = 0,57283$; againſt 57, and under periphery is 2,5114; and under diff. is 131; And 131×283 = 37073, or 0,00037.

Then (2,5114+0,0037×32,54=) 81,841 is the periphery.

PROPOSITION III.

To find the area of an ellipſis; the tranſverſe and conjugate diameters, or axes, being known.

RULE.

Multiply the tranſverſe by the conjugate diameter; the product multiplied by 0,7854 will give the area required.

EXAMPLE.

What is the area of an ellipſis whoſe tranſverſe axe is 24, *and the conjugate* 18?

Then (44×18×0,7854=) 339,2928 is the area.

Note, The area of an ellipſis is a mean proportional between the area of a circle on its tranſverſe axe, and the area of a circle on its conjugate axe.

PRO-

PROPOSITION IV.

In an elliptical ſegment, (PDF,PAH) *the baſe*, (PE, PH) *or double ordinate*; *the height*, (GD,IA=X) *or abſciſs*; *and an abſciſs* (Dg, Ae=*x*) *correſponding to an ordinate* (*pg*,*pe*) *equal to a fourth of the baſe*, (PF,PH) *being known*; *to find the length of* (DK,AB) *that axe of the ellipſe to which the baſe is perpendicular.* Pl. III. Fig. 2.

R U L E. From the ſquare of the greater abſciſs, ſubtract four times the ſquare of the leſſer abſciſs; divide the remainder by the greater abſciſs leſſened by 4 times the leſſer abſciſs; and the quotient will ſhew the axe required.

Or, the axe $=\dfrac{XX-4xx}{X-4x}$

If the difference between the axe and abſciſs, multiplied by the abſciſs, be greater than the ſquare of its correſponding ordinate; the axe found is the tranſverſe; if leſs, the conjugate.

Ex. I. *In a ſegment of an ellipſis, the half baſe, or ordinate is* 40; *the height, or abſciſs, is* 12; *and the abſciſs to the ordinate* 20, *is* 2,50455 : *Required the axe to which the ordinates are perpendicular?*

Then $\left(\dfrac{\overline{12}^2 - \overline{2,50455}^2 \times 4}{12 - 2,50455 \times 4} = 59,99,\text{\&c. or}\right)$ 60 is the axe.

Now $\overline{60-12}\times 12=576$; and $40\times 40=1600$.
Therefore 60 is the conjugate axe.

Ex. II. *In an elliptic ſegment, whoſe abciſs is* 10, *and ordinate* 18; *and the abſciſs to the ordinate* 9, *is* 2,30304; *Required the axe to which theſe ordinates are perpendicular?*

Then $\left(\dfrac{\overline{10}^2 - \overline{2,30304}^2 \times 4}{10 - 2,30304 \times 4} =\right)$ 100 is the axe.

Now $\overline{100-10}\times 10 = 900$; and $18\times 18 = 324$; Therefore 100 is the tranſverſe axe.

PROPOSITION V.

To find the area of an elliptic ſegment, cut parallel to either axe.

I. *The tranſverſe and conjugate axes,* (AB, DK) *and the diſtance* (CF) *of the ſection, parellel to the conjugate, being known.* Fig. 6.

RULE.

In the circumſcribing circle, find the area (*e*B*f*) of the correſponding circular ſegment, (by Prop. XXII. part I.); multiply this area by the conjugate, the product divided by the tranſverſe, will give the area of the elliptical ſegment.

EXAMPLE.

In an ellipſis whoſe tranſverſe diameter (AB) *is* 120; *its conjugate* (DK) 40: *What is the area of a ſegment thereof, cut at the diſtance* (CE) 36, *from the center?*

Now $\left(\frac{120}{2}-36=EB=\right)$ 24 is the height of the

And $\left(\frac{4}{3}-\frac{24\times32}{80\times120-24\times15}\times\sqrt{120\times24\times24}=\right)$ 1610,2488 is the area of the circular ſegment.

Then $\left(1610,2488\times\frac{40}{120}=\right)$536,7496 is the area of the elliptic ſegment.

II. *The tranſverſe and conjugate axes,* (AB,DK) *and the diſtance* (CE) *of the ſection, parallel to the tranſverſe, being known.* Fig. 5.

RULE.

In the inſcribed circle, find the area (*e*D*f*) of the correſponding circular ſegment (by Prop. XXII. part 1.); multiply this area by the tranſverſe; the product divided by the conjuge will give the area of the elliptical ſegment.

EXAMPLE.

In an ellipſis, whoſe tranſverſe axe (AB) *is* 120, *its conjugate* (DK) 40; *What is the area of a ſegment thereof, cut parallel to the tranſverſe, at the diſtance* (CE) 16 *from the center?*

Now $\left(\frac{40}{2} - 16 = ED =\right)$ 4 is the height of the ſeg.

And $\left(\overline{\frac{4}{3} - \frac{4 \times 32}{80 \times 40 - 15 \times 4}} \times \sqrt{40 \times 4 \times 4} =\right)$ 65,398 is the area of the circular ſegment.

Then $\left(65{,}398 \times \frac{120}{40} =\right)$ 196,194 is the area of the elliptical ſegment.

III. *When there are known, the tranſverſe and conjugate axes*; (AB,DK) *and alſo the degrees in the circular arc* (*e* D*f*, *e* B*f*,) *of the inſcribed or circumſcribed circles*), *cut off by the baſe* (2EF) *of the ſegment*, Fig. 4.

RULE.

Multiply the degrees by 0,0174533; from the product take the ſine of the given degrees; (found in tables); multiply the half of the remainder, by half the tranſverſe; the product multiplied by half the conjugate, gives the area required.

EXAMPLE I.

In an ellipſis, whoſe tranſverſe axe (AB) *is* 120; *its conjugate* (DK) 40; *Required the area of a ſegment thereof,* (FDP) *cut parallel to the tranſverſe axe; when the correſponding arc* (*e*D*f*) *of the inſcribed circle contains* 73° 44′ 24″?

Now 73° 44′ 24″ = 73, 74 degrees,
And its natural ſine is 9600015,

$$\text{Th.}\left(\frac{0{,}0174533\times 73{,}74 - 9600015}{2}\times\frac{120}{2}\times\frac{40}{2}=\right)$$

196,2024 is the area.

EXAMPLE II.

In an ellipſis, whoſe tranſverſe axe (AB) *is* 120; *its conjugate* (DK) 40: *Required the area of a ſegment thereof,* (FBL) *cut parellel to the conjugate, when the correſponding arc* (*e*B*f*) *of the circumſcribing circle contains* 106° 15′ 36″?

Now 106° 15′ 36″ = 106,26 degrees,
And its natural ſine is 9600015.

$$\text{Th}\left(\frac{0{,}0174533\times 106{,}26 - 9600015}{2}\times\frac{120}{2}\times\frac{40}{2}=\right)$$

536,7536 is the area.

PROPOSITION VI.

To find the ſolidity of a cylindroid, (or elliptic priſm) the diameters of its end, and its length being known.

RULE.

Multiply the area of the end, by the length, and the product will be the ſolidity required.

EXAMPLE.

What is the ſolidity of a cylindroid, whoſe length is 8 *feet, and the diameters of either end, are* 3 *feet and* 2 *feet?*

Then $(3 \times 2 \times 0{,}7854 \times 8 =)$ 37,6992 is the ſolidity required.

PROPOSITION VII.

To find the convex ſurface of a cylindroid; the length, and the diameters at either end being known.

RULE.

Find (by Prop. II.) the periphery at either end, and this multiplied by the length will give the convex ſurface nearly.

EX-

EXAMPLE.

What is the convex ſurface of a cylindroid whoſe length is 8 feet; and the principal diameters of either end, are 3 feet and 2 feet?

Then $\left(\frac{3+2}{2}\times3{,}1416\times8=\right)$ 62,832 is the convex ſurface.

Or $\frac{2}{3}=,666\cancel{6}$; againſt 66 in the tab. p. 259. is 2,6327, And the diff. $138\times\cancel{6}=90{,}\cancel{8}$ or 91, or 0,0091;

Then $(\overline{2{,}6327+0{,}0091}\times3\times8=)$ 63,4032 is the convex ſurface ſought.

PROPOSITION VIII.

To find the ſolidity of a ſpheroid, the axis of rotation, and the revolving axe being known.

RULE.

Multiply the fixed axe by the ſquare of the revolving axe, the product multiplied by 0,5236 $\left(=\frac{p}{6}\right)$ will give the ſolidity.

EXAMPLE I.

What is the ſolidity of a prolate ſpheroid, whoſe tranſverſe axe is 100, *and its conjugate is* 60?

Then $(100\times\overline{60}^2\times0{,}5236=)$ 188496 is the ſolidity required.

EXAMPLE II.

What is the ſolidity of an oblate ſpheroid, whoſe longeſt axe is 100, *and ſhorteſt axe is* 60?

Then $(60\times\overline{100}^2\times0{,}5236=)$ 314160 is the ſolidity required.

Note, A ſpheroid is $\frac{2}{3}$ of its circumſcribing cylinder.

PROPOSITION IX.

To find the ſuperficial content of a ſpheroid, the tranſverſe and conjugate axes being known.

RULE.

From the ſquare of half the tranſverſe, take the ſquare of half the conjugate; let the ſquare root of the remainder be called A.

In a prolate ſpheroid; multiply 0,0174534 by the degrees correſponding to a ſine, produced by dividing A by half the fix'd axe: Call the product B.

In an oblate ſpheroid; multiply 2,302585 by the common logarithm correſponding to the quotient, of the ſum of A and half the revolving Axe, divided by half the fix'd Axe: Call the product B.

Multiply B by the ſquare of half the fix'd Axe; divide the product by A; to the quotient add half the revolving Axe; the ſum multipled by the revolving Axe, and by 3,1416, will give the ſuperficies required.

EXAMPLE I.

What is the ſuperficial content of a prolate ſpheroid; whoſe longeſt diameter is 100, *and the ſhorteſt is* 60?

Here 100 is the fix'd Axe; and 60 the revolving Axe.

Now $\left(\sqrt{\overline{\tfrac{100}{2}}\Big|^{2}-\overline{\tfrac{60}{2}}\Big|^{2}}=\right)40=A.$

And $(40\div\frac{100}{2}=)$ 0,8, is the ſine of 53,13 degrees nearly.

Then $(\overline{53,13\times0,0174534}=)\,0,927289142=B.$

Th. $\left(\frac{0,927289142\times\overline{\frac{100}{2}}\big|^{2}}{40}+\frac{60}{2}\times60\times3,1416=\right)$ 16579,273 is the ſuperficies ſought.

EXAMPLE II.

What is the ſuperficial content of an oblate ſpheroid, whoſe longeſt diameter is 100, *and the ſhorteſt is* 60?

Here 60 is the fix'd Axe; and 100 the revolving Axe.

Now $\left(\sqrt{\overline{\tfrac{100}{2}}\Big|^{2}-\overline{\tfrac{60}{2}}\Big|^{2}}=\right)40=A.$

And the logarithm of $\left(\overline{40+\frac{100}{2}}\div\frac{60}{2}=\right)3$, is 0,4771213.

Then $(\overline{2,302585\times0,4771213}=)\,1,0986123=B.$

Th. $\left(\frac{1,0986123\times\overline{\frac{60}{2}}\big|^{2}}{40}+\frac{100}{2}\times100\times3,1416=\right)$ 23473,632 is the ſuperficies ſought.

PROPOSITION X.

To find the ſolidity (ABC=S) *of a ſegment of a ſpheroid, cut parallel to either axe; thoſe axes,* (DE=*r*,FB=*f*) *and the height* (*a*B=*h*) *of the ſegment being known.* Pl. III. Fig. 7. 8.

I. *When the ſection* (AC) *is perpendicular to the axis of rotation* (FB).

Here the ſection will be circular.

RULE.

Divide the ſquare of the revolving axe, by the ſquare of the fixed Axe; multiply the quotient by thrice the fixed axe, leſſened by twice the height of the ſegment; multiply the product by the ſquare of the ſaid height; this product multiplied by 0, 5236 will give the ſolidity of the ſegment.

Or $S = \frac{rr}{ff} \times \overline{3f - 2h} \times hh \times \frac{p}{6}$.

EXAMPLE I.

In a prolate ſpheroid, whoſe tranſverſe axe is 100, *its conjugate* 60; *what is the ſolidity of a ſegment thereof, whoſe height is* 10, *and cut perpendicular to the tranſverſe axe?*

Now $\frac{\overline{60}|^{2}}{\overline{100}|^{2}} = 0{,}36$.

And $100 \times 3 - 10 \times 2 = 280$.

Then $(0{,}36 \times 280 \times \overline{10}|^{2} \times 0{,}5236 =)$ 5277, 888 is the ſolidity requird.

EXAMPLE II.

In an oblate ſpheroid, whoſe greater diameter is 100, *its leſſer* 60; *what is the ſolidity of a ſegment thereof, whoſe heighth is* 12, *and cut perpendicular to the conjugate axe?*

Now $\frac{\overline{100}|^2}{\overline{60}|^2} = 2{,}\dot{7}$.

And $60 \times 3 - 12 \times 2 = 156$.

Th. $(2{,}\dot{7} \times 156 \times \overline{12}|^2 \times 0{,}5236 =)\ 32672{,}64$ is the ſolidity required.

II. *When the ſection* (AC) *is parallel to the axis of rotation* (DE).

Here the ſection will be elliptical.

RULE.

Divide the fixed axe by the revolving axe; multiply the quotient by thrice the revolving axe, leſſened by twice the height of the ſegment; multiply the product by the ſquare of the ſaid height; this product multiplied by 0,5236 will give the ſolidity of the ſegment.

Or $S = \frac{f}{r} \times \overline{3r - 2h} \times hh \times \frac{p}{6}$.

EXAMPLE I.

In a prolate ſpheroid, whoſe tranverſe axe is 100; *its conjugate* 60; *what is the ſolidity of a ſegment thereof, whoſe height is* 12; *and cut parallel to the tranverſe axe?*

Now $\frac{100}{60} = 1,\not{6}$.

And $60 \times 3 - 12 \times 2 = 156$.

Then $(1,\not{6} \times 156 \times \overline{12}|^2 \times 0,5236 =)$ 19603,584 is the ſolidity required.

EXAMPLE II.

In an oblate ſpheroid, whoſe longeſt axe is 100; *its ſhorteſt axe is* 60; *what is the ſolidity of a ſegment thereof, whoſe height is* 10; *and cut parallel to the conjugate axe?*

Now $\frac{60}{100} = 0,6$.

And $100 \times 3 - 10 \times 2 = 280$.

Then $(0,6 \times 280 \times \overline{10}|^2 \times 0,5236 =)$ 8796,48 is the ſolidity required.

III. *The ſolidity of a ſpheroidical ſegment may be found by the following,*

RULE.

As the ſolidity of the ſphere $\left\{ \begin{array}{l} \text{circumſcribed,} \\ \text{inſcribed.} \end{array} \right.$

To the ſolidity of the ſpheroid.

So is the ſolidity of a ſpherical ſegment.

To the ſolidity of the correſponding elliptical ſegment.

EXAMPLES.

I. *In a prolate ſpheroid, where the longeſt axe is* 100, *and the ſhorteſt is* 60: *What is the ſolidity of a ſegment thereof, whoſe height is* 10, *and cut perpendicular to the tranverſe axe?*

Now ($\overline{100}|^3 \times 0{,}5236 =$) 523600 is the ſolidity of the circumſcribing ſphere.

And ($\overline{60}|^2 \times 100 \times 0{,}5236 =$) 188496 is the ſolidity of the ſpheroid.

Alſo ($\overline{100 \times 3 - 10 \times 2} \times \overline{10}|^2 \times 0{,}5236 =$) 14660,8 is the ſolidity of the correſponding ſpherical ſegment.

Then (523600 : 188496 : : 14660,8 :) 5277,888 is the ſolidity of the ſpheroidal ſegment required.

II. *In a prolate ſpheroid whoſe tranſverſe axe is* 100, *its conjugate* 60; *what is the ſolidity of a ſegment thereof, whoſe height is* 12; *and cut parallel to the tranſverſe?*

Now ($\overline{60}|^3 \times 0{,}5236 =$) 113097,6 is the ſolidity of the inſcribed ſphere.

And 188496 is the ſolidity of the ſpheroid.

Alſo ($\overline{60 \times 3 - 12 \times 2} \times \overline{12}|^2 \times 0{,}5236 =$) 11762,1504 is the ſolidity of the correſponding ſpherical ſegment.

Then (113097,6 : 188496 : : 11762,1504 :) 19603,584 is the ſolidity of the ſpheroidal ſegment required.

III. *In an oblate ſpheroid, whoſe longeſt diameter is* 100, *its ſhorteſt* 60; *required the ſolidity of a ſegment whoſe height is* 12; *and cut parallel to the longer axe?*

Now the ſolidity of the inſcribed ſphere is 113097,6.

And the ſolidity of the ſpheroid is 314160.

And the ſolidity of the correſponding ſpherical ſegment is 11762,1504.

Then (113097,6 : 314160 : : 11762,1504 :) 32672,64 is the ſolidity of the ſpheroidal ſegment.

IV. *In an oblate ſpheroid, whoſe principal axes are* 100 *and* 60; *required the ſolidity of a ſegment whoſe height is* 10; *and cut parallel to the ſhorteſt axe?*

Now the ſolidity of the circumſcribing ſphere is 523600.

And the ſolidity of the ſpheroid is 314160.

Alſo the ſolidity of the correſponding ſpherical ſegment is 14460,8.

Then (523600 : 314160 : : 14660,8 :) 8796,48 is the ſolidity required.

PRO-

PROPOSITION XI.

To find the ſolidity (S) *of a fruſtum* (DEAC) *of a ſpheroid, one end* (DE) *paſſing thro' the centre* (G) *of the ſpheroid, perpendicular to an axe* (FB); *the diameters* (DE=D, AC=*d*) *of the ends, and their diſtance* (Ga=*h*) *being known.* Pl. III. Fig. 7. 8.

I. *When the ends are perpendicular to the axis of rotation.*

Here each end will be circular.

RULE.

To twice the ſquare of the diameter of the greater end, add the ſquare of the diameter of the leſſer end; multiply the ſum by the diſtance of the ends; the product multiplied by 0,2618 will give the ſolidity required.

Or $S = \overline{2\,DD + dd} \times h \times \frac{p}{12}$.

Ex. I. *What is the ſolidity of a fruſtum of a prolate ſpheroid, the ends being perpendicular to the tranſverſe axe; the diameter of the greater end being* 60, *that of the leſſer end* 36; *and the diſtance of the ends* 40?

Then ($\overline{\overline{60}^2 \times 2 + \overline{36}^2} \times 40 \times 0{,}2618 =$) 88970,112 is the ſolidity ſought.

Ex. II. *What is the ſolidity of the fruſtum of an oblate ſpheroid, the ends being perpendicular to the conjugate axe; the diameter of the greater end being* 100, *that of the leſſer end* 80, *and the diſtance of the ends* 18?

Th. ($\overline{100^2\times 2+80^2}\times 18\times 0{,}2618=$) 124407,36 is the ſolidity ſought.

II. *When the ends are parallel to the axis of rotation.*

Here each end will be elliptical.

RULE.

Multiply twice the tranſverſe diameter of the greater end, by its conjugate; to the product, add the rectangle under the tranſverſe and conjugate diameters of the leſſer end; multiply the ſum by the diſtance of ends, the product multiplied by 0,2618 will give the ſolidity required.

Or putting T,C, for the longeſt and ſhorteſt diameters of the greater end.

t,c, thoſe of the leſſer end.

Then $S=\overline{2\,TC+tc}\times h\times\frac{p}{12}$.

Ex. III. *In the fruſtum of a prolate ſpheroid, the ends being parallel to the tranſverſe axe; the two diameters of the greater end, are* 100; 60; *and the two diameters of the leſſer end, are* 80; 48; *the diſtance of the ends is* 18: *Required the ſolidity of that fruſtum?*

Then ($\overline{2\times 100\times 60+80\times 48}\times 18\times 0{,}2618=$) 4644,416 is the ſolidity ſought.

Ex. IV. *In the fruſtum of an oblate ſpheroid, the ends being parallel to the conjugate axe; the two diameters of the greater end are* 100; 60; *the two diameters of the leſſer end, are* 60; 36; *and the diſtance of the ends is* 40; *required the ſolidity of that fruſtum?*

Then ($\overline{2\times100\times60+36\times60}\times40\times0,2618=$) 148283,52 is the ſolidity ſought.

PROPOSITION XII.

To find the ſuperficial content of a fruſtum of a ſpheroid, one of whoſe parallel ends, paſſes thro' the centre of the ſpheroid, perpendicular to the fix'd axe, the diameters of the ends of the fruſtum, and their diſtance being known.

RULE.

1. Find the fix'd axe.

2. Let the ſquare root, of the difference of the ſquares, of half the fix'd, and half the revolving axe, be called A.

3. Multiply the ſquare of A, by the ſquare of the length; $\left\{\begin{matrix}\text{take}\\ \text{add}\end{matrix}\right\}$ the product $\left\{\begin{matrix}\text{from}\\ \text{to}\end{matrix}\right\}$ the fourth power of half the fix'd axe; let the ſquare root of the $\left\{\begin{matrix}\text{remainder}\\ \text{ſum}\end{matrix}\right\}$ be called B.

4. In a prolate ſpheroid; multiply A, by the length of the fruſtum; divide the product by the ſquare of half the longeſt axe; ſeek the quotient among the ſines; multiply the correſponding degrees by 0,0174534: Call the product D.

4. In an oblate ſpheroid: Multiply A by the length; to the product add B; divide the ſum by the ſquare of half the ſhorteſt axe; multiply the logarithm of the quotient by 2,302585: Call the product D.

5. Divide the ſquare of half the fix'd axe by A; multiply the quotient by D: Alſo, divide the length by the ſquare of half the fix'd axe; multiply the quotient by B: The ſum of the two products, multiplied by half the revolving axe, and by 3,1416, will give the ſuperficies required.

Note, The difference between the convex ſurface of the hemiſpheroid and this fruſtum, will give the convex ſurface of a ſegment of a ſpheroid.

EXAMPLES.

I. *What is the convex ſurface of a fruſtum of a prolate ſpheroid, the diameter of the leſſer end being* 36, *that of the greater end, or revolving axe,* 60; *and the diſtance* 40?

Now $\sqrt{\left.\overline{\frac{60}{2}}\right|^2 - \left.\overline{\frac{36}{2}}\right|^2} = 24.$

And $\frac{\frac{1}{2}60 \times 40}{24} \times 2 = 100$ is the fix'd axe.

And $\sqrt{\left.\overline{\frac{100}{2}}\right|^2 - \left.\overline{\frac{60}{2}}\right|^2} = 40 = A.$

Alſo $\frac{40 \times 40}{50 \times 50} = 0{,}64$ is the ſine of 39,79 degrees.

Then $(39{,}79 \times 0{,}0174534 =)\ 0{,}69447 = D.$

And $(\sqrt{\overline{50}|^4 - \overline{40}|^2 \times \overline{40}|^2} =)\ 1920{,}94 = B.$

Then $\left(\frac{50 \times 50}{40} \times 0{,}69447 =\right)\ 43{,}404375 =$ one product.

And $\frac{40}{50 \times 50} \times 1920{,}94 = 30{,}7349968 =$ other product.

Therefo. $43{,}404375 + 30{,}734968 = 74{,}139372.$

Then $(74{,}139372 \times \frac{60}{2} \times 3{,}1416 =)\ 6987{,}518947$ is the ſurface required.

II. *What is the convex ſurface of a fruſtum of an oblate ſpheroid, the diameter of the leſſer end being* 80, *that of the greater end, or revolving axe,* 100; *and the diſtance of the ends* 18?

Now $\sqrt{\overline{\tfrac{100}{2}}\Big|^{2} - \overline{\tfrac{80}{2}}\Big|^{2}} = 30.$

And $\left(\frac{\frac{1}{2}\,100 \times 18}{30} \times 2 =\right)$ 60 is the fix'd axe.

Then $\left(\sqrt{\overline{\tfrac{100}{2}}\Big|^{2} - \overline{\tfrac{60}{2}}\Big|^{2}} =\right) 40 = A.$

And $\left(\sqrt{\overline{30}|^{4} + \overline{40}|^{2} \times \overline{18}|^{2}} =\right) 1152{,}5624 = B.$

Alſo $\frac{40 \times 18 + 1152{,}5264}{30 \times 30} = 2{,}080625$; whoſe log. is 0,3181938.

Theref. $(0{,}3181938 \times 2{,}302585 =)\ 0{,}7326682 = D.$

Now $\left(\frac{30 \times 30}{40} \times 0{,}7326682 =\right) 16{,}4850345$ = one product.

And $\frac{18}{30 \times 30} \times 1152{,}5624 = 23{,}051248 =$ other product.

Alſo $16{,}4850345 + 23{,}051248 = 39{,}5362825.$

Then $\left(39{,}5362825 \times \frac{102}{2} \times 3{,}1416 =\right)$ 6210,359254 is the ſurface required.

PROPOSITION XIII.

In an elliptical ſpindle (KDLA) *the length or axis* (KL=*l*) ; *a perpendicular diameter* (DA=2D) *in the middle* ; *and another parallel thereto* (EH=2*d*) *biſecting the half length, being known* ; *to find the axes* (FB=2*t*, GD =2*c*) *of the generating ellipſis.* Pl. III. Fig. 9.

RULE.

From the ſquare of half the greater diameter, take the ſquare of the difference of the two diameters ; divide the remainder by half the greater diameter, leſſened by twice the difference of the two diameters ; the quotient will be that diameter of the ellipſis, to which the axe of the ſpindle is perpendicular.

Or $\frac{DD-\overline{D-d}^2}{D-\overline{2D-2d}}$ = an axe of the ellipſe.

And the tranſverſe will be found by caſe III. prop. I.

Ex. *In an elliptical ſpindle, whoſe length is* 80 ; *the greater diameter* 24 ; *and the diameter at a quarter the length is* 18,99094 : *Required the tranſverſe and conjugate axes of the generating ellipſe?*

Now 24—18,99094 = 5,00906 the difference of the two diameters,

Then $\left(\frac{\overline{12}^2-\overline{5{,}00906}^2}{12-10{,}01812}\right.$ = 59,99, &c. or) 60 is the conjugate axe.

Hence the tranſverſe will be 100.

In the fruſtum of an elliptical ſpindle, where one end paſſes thro' the centre of the ſpindle; the length of the fruſtum; the diameters of the ends; and a diameter equally diſtant from the ends, being known; the axes of the ellipſe will be found as follows.

RULE. From the ſquare of half the difference, of the greater and leſſer diameters, take the ſquare of the difference of the greater and mean diameters for a dividend. From half the difference of the greater and leſſer diameters, take twice the difference of the greater and mean diameters for a diviſor; the quotient will be that axe of the ellipſe, to which the ſection is perpendicular.

Ex. *In the fruſtum of an elliptical ſpindle (parallel to the tranſverſe) whoſe length is* 14; *the diameter of the greater end* 24; *that of the leſſer end* 21,6; *and the diameter in the midway* 23,40909; *What are the axes of the ellipſe?*

Now $\left(\frac{24}{2} - \frac{21,6}{2} =\right)$ 1,2 is the half difference of the greater and leſſer diameters.

And (24—23,40909=) 0,59091 is the difference between the greater and mean diameters.

Then $\left(\frac{\overline{1,2}|^{2} - \overline{0,59091}|^{2}}{1,2 - 0,59091 \times 2} =\right)$ 60 is the conjugate axe.

And 100 is the tranſverſe axe.

PROPOSITION XIV.

To find the ſolidity (S) *of an elliptical ſpindle* (KDLA); *wherein the length of its axe* (KL=*l*); *its greateſt diameter* (AD=D); *and the diameter* (EH) *at a quarter of the length from one end are known; the axe of the ſpindle being parallel to that of the generating ellipſis.* Pl. III. Fig. 9.

RULE.

1. Find the axes of the ellipſe, and the central diſtance (GI=*a*).

2. Find the area (A) of an elliptic ſegment, the meaſures of whoſe baſe and height, are reſpectively equal to thoſe of the axe and half diameter of the ſpindle.

3. Divide this area, by a third of the ſpindle's length; ſubtract the quotient from the ſpindle's diameter; multiply the remainder by eight times the central diſtance; to the product add twice the ſquare of the given diameter; multiply the ſum by the given length; the product multiplied by 0,2618 will give the ſolidity required.

Or $S=\overline{\overline{\overline{D-\frac{A}{\frac{1}{3}l}}\times 8a+2DD}\times l}\times\frac{p}{12}$.

EX-

EXAMPLE.

What is the ſolidity of an elliptical ſpindle, the length of whoſe axe is 80; the greateſt diameter is 24; and the diameter at 20, from the end, is 18,99694?

Now the conjugate and tranſverſe diameters of the ellipſe, are 60 and 100.

Then $\left(\frac{60}{2} - \frac{24}{2} =\right)$ 18 is the diſtance of the centers.

Alſo the area of the elliptic ſegment, whoſe baſe is 80, and height 12; will be found equal to 670,942.

Then $\frac{670,942}{80\div 3} = 25,160325$

And $24 - 25,160325 = -1,160325$

Therefore $-1,160325 \times 8 \times 18 = -167,0868$.

Then $(\overline{2\times\overline{24}|^2 - 167,0868} \times 80 \times 0,2618 =)$ 20628,022 is the ſolidity ſought.

PROPOSITION XV.

To find the ſolidity S *of a fruſtum* (DAHE) *of an elliptical ſpindle, one of whoſe parallel ends* (AD) *paſſes thro' the centre* (I) *of the ſpindle: The diameters* (AD=D,EH=d), *of thoſe ends, their diſtance* (IB=l), *and a diameter* (ef) *taken in the midway, being known.* Pl. III. Fig. 9.

RULE.

Find the axes of the ellipſe and the central diſtance (GI=a).

Find the area (A) of an elliptical ſegment, whoſe baſe is twice the length of the fruſtum; and whoſe height is equal to the difference of half the diameters of the ends.

Divide half this area by a third of the length; to the quotient add the leſſer diameter; ſubtract the ſum from the greater diameter; multiply the remainder by eight times the central diſtance; to the product add the ſum of the ſquare of the leſſer diameter, and twice the ſquare of the greater diameter; multiply the ſum by the given length; the product mnltiplied by 0,2618 will give the ſolidity required.

$$\text{Or } S = \overline{\overline{\overline{D - \overline{\frac{\frac{1}{2}A}{\frac{1}{3}l} + d}} \times 8a} + 2DD + dd} \times l \times \frac{p}{12}.$$

EX-

EXAMPLE.

What is the ſolidity of a fruſtum of an elliptical ſpindle, whoſe length is 14; *the diameter at the greater end, is* 24; *that at the leſſer end is* 21,6; *and a diameter in the midway, is* 23,40909?

Now 60 and 100 will be found for the conjugate and tranſverſe axes of the ellipſe.

And $\left(\frac{60}{2}-\frac{24}{2}=\right)$ 18 is the central diſtance.

Alſo 22,48937 is the area of an elliptical ſegment, whoſe height is 1,2 and baſe 28.

Its half is 11,24468.

Then $\frac{11,24468}{14 \div 3}+21,6=24,00957.$

And $\overline{24-24,00957} \times 8 \times 18=-1,37808.$

Alſo $2 \times \overline{24}|^2+\overline{21,6}|^2=1618,56.$

Then $(\overline{1618,56-1,37808} \times 14 \times 0,2618=)$ 5927,29515 is the ſolidity required.

SECTION II.

Of parabolic lines, ſuperficies, and ſolids.

PROPOSITION XVI.

To find the area of a parabola, the double ordinate (or baſe CD*), and axis (or height* AB*) being known.* Pl. III. Fig. 10.

RULE.

Multiply the baſe by the height; and two thirds of this product will be the area required.

EXAMPLE.

What is the area of a parabola, whoſe axis is 12; *and the double ordinate is* 16?

Th. ($16 \times 12 \times \frac{2}{3} =$) 128 is the area.

Note, Every conical parabola is $\frac{2}{3}$ of its circumſcribing parallelogram.

PROPOSITION XVII.

To find the area (A) *of a fruſtum* (DHGC) *of a parobola, whoſe parallel ends* (DC=B, GH=*b*), *and their diſtance* (IB=*d*) *are known.* Pl. III. Fig. 10.

RULE.

To the ſquare of the greater end, add the ſquare of the leſſer end, and the product of the ends; divide the ſum, by the ſum of the ends; the quotient multiplied by two thirds of the diſtance of the ends will give the area ſought.

$$\text{Or } A = \frac{BB+bb+Bb}{B+b} + \tfrac{2}{3}d.$$

EXAMPLE

Suppoſe the end CD *is* 24; *the end* GH *is* 20; *and the diſtance* IB, $5\frac{1}{2}$: *Required the area* CGHD?

$$\text{Then } \left(\frac{\overline{24}|^2+\overline{20}|^2+24\times 20}{24+20}\times 5{,}5\times\tfrac{2}{3}=\right) 121{,}3 \text{ is}$$

is the area required.

PROPOSITION XVIII.

To find the length (L) *of the curve of a papabola, whoſe abſciſs* (AB$=x$), *and ordinate* (CB$=y$) *are known.* Pl. III. Fig. 10.

RULE.

Divide the ſquare of the ordinate by twice the abſciſs, the quotient is the half parameter.

$$\text{Or } p=\frac{yy}{2x}.$$

To the ſquare of the half parameter add the ſquare of the ordinate, the ſquare root of the ſum, call A.

Or $A = \sqrt{pp+yy}$.

Multiply A by the ordinate, divide the ſum by half the parameter; call the quotient B.

Or $B = \frac{yA}{p}$.

Add A to the ordinate, divide the ſum by half the parameter, ſeek the tabular logarithm to the quotient; multiply the logarithm by 2,302585; the product multiplied by half the parameter, and B added to the product, gives the length of the curve required.

Or $L = B + 2{,}302585 \times \log. \frac{A+y}{p}$.

EXAMPLE.

What is the length of the curve of a parabola, whoſe abſciſs is 50, and ordinate 30?

Now $\left(\frac{\overline{30}|^2}{50 \times 2} =\right)$ 9 is the half parameter.

And $\left(\sqrt{\overline{9}|^2 + \overline{30}|^2} =\right) 31{,}321 = A$.

Alſo $\left(\frac{31{,}321 \times 30}{9} =\right) 104{,}403 = B$.

But $\frac{31{,}321+30}{9} = 6{,}8134$.

Its logarithm $= 0{,}8333639$.

Then $(\overline{0{,}8333639 \times 2{,}302585 \times 9} + 104{,}403333$ $=)$ 121,673352 is the length of the curve required.

PROPOSITION XIX.

*To find the ſolidity of a parabolic conoid (*ABD, A*af); the diameter* (BD, *or fa, mn) of the baſe, and the height* (AC *or rs) being known.* Pl. III. Fig. 11.

RULE. Multiply the ſquare of the diameter of the baſe, by the height; then the product multiplied by 0,3927 will give the ſolidity.

Or, multiply the area of the baſe by half the height, and the product will be the ſolidity.

Note, A paraboloid is half of its circumſcribing cylinder.

EXAMPLES.

I. *What is the ſolidity of a paraboloid, whoſe height is* 50; *and the diameter of its circular baſe is* 60?

Then ($\overline{60}|^2 \times 50 \times 0{,}3927 =$) 70686 is the ſolidity required.

II. *In the ſegment of a paraboloid, the greater and leſſer diameters of the elliptic baſe are* 300 *and* 60; *and the height is* 10: *Required the ſolidity of that ſegment?*

Then ($300 \times 60 \times 0{,}7854 \times 10 =$) 70686 is the ſolidity ſought.

PRO-

PROPOSITION XX.

*To find the convex ſuperficies (s) of a parabolic conoid (*ABD*); the abſciſs (*AC$=x$*) and ordinate (*BC$=$CD$=y$*) of the generating parabola being known* Pl. III. Fig. 11.

RULE.

To four times the ſquare of the abſciſs, add the ſquare of the ordinate; let the ſquare root of the ſum be called A. Or A $= \sqrt{4xx+yy}$.
To A, add the ordinate; let their ſum be a diviſor to the ſquare of the ordinate; to the quotient add A; multiply the ſum by the ordinate; the product multiplied by 2,0944, will give the convex ſuperficies required.

Or $s = \overline{A + \frac{yy}{A+y}} \times y \times \frac{2}{3}p$.

EXAMPLE.

What is the convex ſuperficies of a paraboloid, the diameter of whoſe baſe is 60, *and the height* 50? *Or the ordinate* 30, *and abſciſs* 50?

Now ($\sqrt{4\times50\times50+30\times30}=$) 104,403 $=$ A.
And ($\frac{900}{104,403+30}=$) 6,69628 is the quotient.
Alſo 6,69628 $+$ 104,403 $=$ 111,09928.
Then (111,09928 $\times$ 30 $\times$ 2,0944 $=$) 6980,58996 is the convex ſuperficies required.

PROPOSITION XXI.

To find the ſolidity (S) *of a fruſtum* (BD*db*) *of a paraboloid, contained between two parallel planes, each perpendicular to the axe* (AC); *the diameters* (BD=D, *bd*=*d*) *at thoſe ſections, and the diſtance* (C*c*=*h*) *of the ends being known.*

RULE.

To the ſquare of the diameter of the greater end, add the ſquare of the diameter of the leſſer end; multiply the ſum by the length, or height; and the product multiplied by 0,3927 will give the ſolidity.

Or $S = \overline{DD + dd} \times h \times \frac{1}{8}p$.

EXAMPLE.

What is the ſolidity of a parabolic fruſtum, the diameter of the greater end being 60, *that of the leſſer end* 48, *and the diſtance of the ends being* 18?

Then $(\overline{\overline{60}^2 + \overline{48}^2} \times 18 \times 0,3927 =)$ 41733,0144 is the ſolidity required.

PROPOSITION XXII.

In a paraboloid (ABD) *whoſe baſe* (B*b*D*d*) *is perpendicular to the axe* (AC). *To find the ſolidity* (S) *of a ſegment thereof,* (*a*D*bd*) *cut parallel to the axe; the abſciſs* (*ac*=*h*) *the ordinate* (*dc*=*cb*=*y*) *at the ſection, and alſo the height* (D*c*=*x*) *of the circular ſegment of its end being known.* Pl. III. Fig. 11.

RULE.

RULE.

Find the radius (r) of the baſe of the paraboloid; divide the ſquare of this radius by the ſquare of the ordinate; multiply the quotient by half the circular ſegment, (bDd=2A) at the end of the conoidal ſegment; (ſound prop. XXII. part 1.) from the product ſubtract one third of the ordinate, multiplied by the diſtance thereof from the axe; the remainder multiplied by the abſciſs, or length of the conoidal ſegment, will give the ſolidity required.

$$\text{Or } S = \overline{\frac{rr}{yy}\times A - \frac{1}{3}\, y \times \overline{r - x}} \times b.$$

EXAMPLE.

In a ſlice (aDbd) *cut from a paraboloid parallel to the axe, and perpendicular to the baſe thereof; the abſciſs* (ac) *is* 18; *the ordinate* (dc=cb) *is* 18; *and the height* (Dc) *of the circular ſegment at the end is* 6: *Required the ſolidity of the ſlice, or conoidal ſegment?*

Now $\left(\frac{18\times18+6\times6}{2\times6} =\right)$ 30 is the radius.

And 147,35298 is the area of the circular ſegment bDd; its half is 73,67649.

Then $\overline{\frac{30\times30}{18\times18}\times 73{,}67649 - \frac{18\times\overline{30-6}}{3}} \times 18 =)$

1091,824488 is the ſolidity of the ſegment.

PROPOSITION XXIII.

To find the ſolidity (S) *of a parabolic ſpindle* (BACD), *the axis of rotation* (BC$=l$), *and the greateſt diameter* (AD$=$D) *of the ſolid being known.* Pl. III. Fig. 13.

RULE.

Multiply the ſquare of the diameter by the length, the product multiplied by 0,418879 will give the ſolidity.

Or $S = DD \times l \times \frac{2}{15} p$.

EXAMPLE.

Suppoſe the length of a parabolic ſpindle be 9 *feet, and the greateſt diameter is* 3 *feet*: *Required the ſolidity?*

Then $(3 \times 3 \times 9 \times 0{,}418879 =)$ 33,929199 is the ſolidity required.

Note, A parabolic ſpindle is $\frac{2}{15}$ of its circumſcribing cylinder.

PROPOSITION XXIV.

To find the ſolidity (S) *of a fruſtum, or zone,* (AFED) *of a parabolic ſpindle, contained between two parallel ends* (AD,FE) *the greater whereof* (AD) *paſſes through the middle* (I) *of the ſpindle, perpendicular to the axe* (BC). *The diameters* ($AD=D, FE=d$) *of the ends, and their diſtance* ($IG=l$) *being known.*

RULE.

To eight times the ſquare of the greater diameter, add thrice the ſquare of the leſſer diameter, and four times the product of the two diameters; multiply the ſum by the diſtance of the ends, the product multiplied by 0,05236 will give the ſolidity.

Or $S=\overline{8DD+3dd+4Dd}\times l\times\frac{p}{6}$

EXAMPLE.

What is the ſolidity of a fruſtum, or zone of a parabolic ſpindle, the diameter of the greater end being 36 *inches, that of the leſſer end* 20 *inches, and the diſtance of the ends* 36 *inches?*

Now $36\times36\times8 = 10368$

And $20\times20\times3 = 1200$

Alſo $36\times20\times4 = 2880$

Their ſum is 14448

Then $14448\times36\times0{,}05236=27233{,}9$ is the ſolidity required.

SECTION III.

Of hyperbolic lines, ſuperficies and ſolids.

In elliptic and parabolic figures, the lines contained, or drawn within them, are ſufficient to determine moſt of the problems that can be propoſed concerning their ſuperficies, ſolidities, *&c.* But ſuch things cannot ſo conveniently be done in the hyperbola, without knowing the relation which ſome lines within the figure have to others that lie without it, and of theſe the moſt uſeful are the tranſverſe and conjugate diameters. In Pl. III. Fig. 14.

The curves GAH, IBK, are hyperbolas; where GH, IK, are double ordinates, AF, BL, are abſciſſas; AB is the tranſverſe axe, DE its conjugate; the lines C*a*, drawn thro' the center C, are called aſſymptotes.

PROPOSITION XXV.

To find the tranſverſe and conjugate axes (AB$=2t$, DE$=2c$) *of an hyperbola, whoſe abſciſs* (AF$=x$), *and ordinate* (GF$=y$), *are known; and alſo the length of the abſciſs* (A$f=z$) *correſponding to an ordinate* ($gf=v$) *equal to half the given one.* (*viz.* $gf=\frac{1}{2}$GF). Pl. III. Fig. 14.

RULE I.

From the ſquare of the greater abſciſs, take 4 times the ſquare of the leſſer abſciſs; divide the remainder by four times the leſſer abſciſs leſſened by the greater abſciſs; the quotient will ſhew the length of the tranſverſe ſought.

Or $2t = \dfrac{xx - 4zz}{4z - x}$.

EX-

EXAMPLE.

What is the tranſverſe axe of an hyperbola, whoſe abſciſs is 40, *ordinate* 48; *and the abſciſs correſponding to an ordinate of* 24, *is* 12,111?

Then $\left(\frac{\overline{40}^2 - 4\times\overline{12,111}^2}{4\times12,111 - 40} = \right)$ 120 is the tranſverſe axe required.

RULE II.

Multiply the abſciſs by the ſum of the tranſverſe and abſciſs; let the ſquare root of product be a diviſor to the product of the tranſverſe and ordinate; the quotient will be the conjugate required.

Or $2c = \frac{2ty}{\sqrt{x \times \overline{2t+x}}}$

EXAMPLE.

Where the tranſverſe is 120, *the abſciſs* 40, *and the ordinate* 48: *What is the conjugate?*

Now $\left(\sqrt{\overline{120+40}\times 40} = \right)$ 80 is the diviſor.

Then $\left(\frac{120\times48}{80} = \right)$ 72 is the conjugate ſought.

PROPOSITION XXVI.

To find the tranſverſe axe $(=2t)$ *of an hyperbola, wherein are known, three equidiſtant ordinates;* $(GF=y,\ be=s,\ gf=v)$, *and alſo, their diſtance* $(Fe=ef=d)$ *from each other.* Plate III. Fig. 14.

RULE.

1. Let the difference of the ſquares of the mean and leſſer ordinates, be called B.

Or put $B = ss - vv$.

2. And the difference of the ſquares of the greater and leſſer ordinates, be called D.

Or put $D = yy - vv$.

3. From four times B, take D, for a dividend; and from D, take twice B for a diviſor: Let the quotient be called A.

Or put $A = \frac{4B-D}{D-2B}$.

4. To A, add 1; multiply the ſum by the ſquare of the leſſer ordinate; divide the product by B; ſubtract the quotient from the ſquare of half A; the ſquare root of the remainder multiplied by the common diſtance of the ordinates, will give half the tranverſe axe.

Or $t = d\sqrt{\overline{\tfrac{1}{2}A}^2 - \frac{\overline{A+1}\times vv}{B}}$.

EX-

EXAMPLE.

Let the greater ordinate be 48; *the leſſer* 27; *the mean ordinate* 38,2132196; *and their common diſtance* 12,5; *required the tranſverſe axe of this hyperbola?*

Now $(\overline{38,2132196}|^2 - \overline{27}|^2 =) 731,250152 = B.$

And $(\overline{48}|^2 - \overline{27}|^2 =) 1575 = D.$

Alſo $\left(\frac{4 \times 731,250152 - 1575}{1575 - 2 \times 731,250152} =\right) 12 = A.$

Then $\frac{\overline{12 + 1} \times \overline{27}|^2}{731,250152} = 12,96.$

And $\left(\sqrt{\overline{\frac{12}{2}}|^2 - 12,96 \times 12,5} =\right) 60$ is the half tranſverſe ſought.

PROPOSITION XXVII.

In an hyperbola, whoſe tranſverſe axe $(=t)$ *conjugate axe* $(=c)$, *and abſciſſa* $(=x)$, *being known; to find the area.*

RULE.

RULE.

To the tranſverſe add $\frac{5}{7}$ of the abſciſs, multiply the ſum by the abſciſs, and take 21 times the ſquare root of the product; call this A.

Or put $A=21\sqrt{t+\frac{5}{7}x\times x}$.
To A add four times the ſquare root of the product of the tranſverſe and abſciſs; call this B.

Or put $B=A+4\sqrt{tx}$.
Divide the conjugate by the tranſverſe; multiply the quotient by $\frac{4}{75}$ of the abſciſs; the product multiplied by B gives the area.

Or area $=\frac{c}{t}\times\frac{4}{75}x\times B$.

Ex. *Suppoſe the tranſverſe is* 100, *the conjugate* 60, *and the abſciſs* 50: *Required the area of the hyperbola?*

Now $(21\times\sqrt{100+\frac{5}{7}\times50\times50}=)$ 1729,88445 = A.
And $(1729{,}88445+4\times\sqrt{100\times50}=)$ 2012,72717 = B.
Then $\left(\frac{60}{100}\times\frac{4\times50}{75}\times2012{,}72717=\right)$ 3220,363472 is the area required.

PRO-

PROPOSITION XXVIII.

To find the ſolidity (S) *of a hyperbolic conoid, the height, (or length* $=x$*) the diameter of the baſe (or end* $=$ D*) and the tranſverſe diameter* ($=t$) *of the generating hyperbola being known.*

RULE.

To thrice the tranſverſe, add twice the height; divide the ſum by the ſum of the tranſverſe and height; multiply the quotient by the height; the product multiplied by the ſquare of the diameter, and by 0,1309, will give the ſolidity.

Or $S = \frac{3t+2x}{t+x} \times x \times DD \times \frac{p}{24}$.

EXAMPLE.

What is the ſolidity of a hyperboloid, whoſe height (or length) is 50; *the diameter of the baſe (or end) is* 103,923048; *and the tranſverſe axe is* 100?

No. $\left(\frac{100\times3+50\times2}{100+50} = \frac{8}{3} =\right)$ $2{,}\cancel{6}$ is the quotient.

And $(\overline{103{,}923048}^{\,2} =)$ 10799,99977, or 10800 is the ſquare of the diameter.

Then $(2{,}\cancel{6}\times50\times10800\times0{,}1309 =)$ 188496 is the ſolidity required.

PROPOSITION XXIX.

To find the convex ſurface (s) of a hyperboloid, the diameetr (D) of whoſe baſe and the height (h) are known: and alſo, the diſtance from the vertex, where the diameter is equal to half that of the baſe.

RULE.

1. Find the tranſverſe ($=2t$) and conjugate ($=2c$) axes.

2. Let the ſquare root, of the ſum of the ſquares, of the half tranſverſe and half conjugate be called A. Or put $A=\sqrt{tt+cc}$.

3. Multiply the ſquare of the ſum, of the height and half tranſverſe, by the ſquare of A; from the product take the fourth power of the half tranſverſe; let the ſquare root of the remainder be called B.

Or put $B=\sqrt{AA\times\overline{t+x}^2-t^4}$.

4. Multiply the ſum of A, and the half conjugate, by the half tranſverſe, for a dividend: Multiply A by the ſum of the height and half tranſverſe; let the product added to B be a diviſor: Multiply the log. of the quotient by 2,302585; call the product C.

Or put $C=2,302585\times\log.\dfrac{\overline{A+c}\times t}{B+\overline{t+x}\times A}$

5. Divide the ſquare of the half tranſverſe by A, multiply the quotient by C; divide the ſum of the height and half tranſverſe by the ſquare of the half tranſverſe, multiply the quotient by B: From the ſum of the two products take the half conjugate; the

remainder multiplied by the half conjugate, and the product by 3,1416, will give the convex ſurface required.

Or $s=\overline{\frac{tt}{A}\times C+\frac{t+x}{tt}\times B-c}\times c\times p$.

EXAMPLE.

What is the ſuperficial content of a hyperboloid; the diameter of whoſe baſe is 48; the height 40; and the diſtance of the vertex, from the place of the diameter 24; is 12,111?

By (pr. 25.) the tranſverſe is 120; and the conjugate is 72.

Now $\left(\sqrt{\overline{\frac{120}{2}}\Big|^{2}\times\overline{\frac{72}{2}}\Big|^{2}}=\right)69{,}97142=A.$

And $\left(\sqrt{\overline{60+40}^{\,2}\times\overline{69{,}97142}^{\,2}-\overline{60}\big|^{4}}=\right)$ 6000 $=$ B.

Alſo $\left(\frac{\overline{69{,}97142+36}\times 60}{\overline{40+60}\times 69{,}97142+6000}=\right)0{,}4892064$;

whoſe log. is $\bar{1},6894922$.

Then $(\bar{1},6894922\times 2{,}302585=)\ \bar{1},2850296=C$ $=-0{,}7149704$.

Again, $\left(\frac{\overline{36}^{2}}{69{,}97142}\times-0{,}7149709=-36{,}77473\right.$.

And $\frac{40+60}{60\times 60}\times 6000=166{,}6$.

Now, $166{,}6-36{,}77473-36=93{,}89193$.

Then $93{,}89193\times 36\times 3{,}1416=10618{,}9518$ is the convex ſurface ſought.

PRO-

PROPOSITION XXX.

To find the ſolidity (S) *of a fruſtum of a hyperbolic conoid; the diameters of the ends,* (*viz.* 2D=*greater* 2*d*=*leſſer*) *and their diſtance* (=*h*) *being known; and alſo, a diameter taken in the midway between the ends.*

RULE.

Find the tranſverſe axe (=2*t*) and the conjugate (=2*c*).

Divide the ſquare of the half conjugate by the ſquare of the half tranſverſe; multiply the quotient by a third of the ſquare of the diſtance of the ends; ſubtract the product from the ſum of the ſquares of the half diameters of the ends; the remainder multiplied by the diſtance of the ends, and the product by 1,5708, will give the ſolidity of the fruſtum required.

$$\text{Or } S=\overline{DD+dd-\frac{cc}{tt}\times\frac{1}{3}hh}\times h\times\frac{p}{2}.$$

EXAMPLE.

What is the ſolidity of a fruſtum of a hyperbolic conoid; whoſe greater diameter is 96; leſſer diameter 54; middle diameter is 76,4264392; and the common diſtance of theſe diameters is 12,5?

Now 60 is the half tranſverſe,

And 36 is the half conjugate,

Then $\frac{\overline{36}|^{2}}{\overline{60}|^{2}}\times\frac{\overline{25}|^{2}}{3}=75$.

And $\left(\overline{\frac{96}{2}}\right|^{2}+\overline{\frac{54}{2}}\Big|^{2}=)$ 3033 is the ſum of the ſquares of the half ends.

Then $\overline{(3033-75}\times25\times1{,}5708=)$ 116160,66 is the ſolidity ſought.

SCHOLIUM.

The ſuperficial contents of ſome of the ſolids in the three foregoing ſections are omitted, becauſe they did not appear to be reducible to eaſy practical rules: Beſide, a multitude of other problems might have been added, concerning the ſolids that can be produced by the rotation of the conic ſections about their axes, abſciſſes, ordinates, tangents, and aſſymptotes: But as ſuch problems (and indeed ſeveral in this work) ſeem to be of little more uſe than the exercise of the elements, by which they may be computed; therefore 'tis more proper to ſeek for them among the treatiſes of exhauſtions, indiviſibles, infinites, and fluxions; particularly the latter, which is moſt in uſe; being far more extenſive than either of the former, and beſt ſuited to difficult enquiries in mathematical ſubjects.

SECTION IV.

Of the ſolidity and ſuperficies of cylindric rings.

DEFINITION.

If a cylinder be circularly bent, until its ends meet, the figure thus formed may be called a cylindric ring.

Or this ſolid may be conceived to be generated by the rotation of a circle, about a right line, as an axe, either touching the circle, or at a given diſtance from it.

A great variety of ſolids may be conceived to be thus generated from different planes, ſuch as elliptic, parabolic, hyperbolic, *&c.*; but in this work, no other will be conſidered but that which ariſes from a circle.

By thickneſs is to be underſtood the diameter of the generating circle.

The inner diameter is twice the diſtance of the axe from the generating circle.

PROPOSITION XXXI.

To find the ſolidity of a cylindric ring, whoſe thickneſs, and inner diameter, are known.

RULE.

To the thickneſs of the ring add the inner diameter; multiply the ſum by the ſquare of the half thickneſs; the product multiplied by 9,8696044 (=*pp*) will give the ſolidity ſought.

EXAMPLE.

What is the ſolidity of an anchor ring, whoſe inner diameter is 8 *inches, and thickneſs in metal* 3 *inches?*

Then $(\overline{8+3} \times \tfrac{3}{2}|^{2} \times 9{,}8696044 =) 488{,}5454$ is the ſolidity of that ring.

When the inner diameter is nothing; this rule will alſo give the ſolidity.

PRO-

PROPOSITION XXXII.

To find the convex ſuperficies of a cylindric ring, whoſe thoſe thickneſs and inner diameter are known.

RULE.

To the thickneſs of the ring add the inner diameter; multiply the ſum by the thickneſs; the product multiplied by 9,8696044, will give the ſolidity required.

EXAMPLE.

A jeweller wonld have for his ſign, a cylindric ring whoſe outſide diameter ſhall be 18 *inches, and* 3 *inches thick: What will the gilding of this ring come to at a penny an inch?*

Now $18 - 3 \times 2 = 12 =$ inner diameter.

Then ($\overline{3 + 12} \times 3 \times 9{,}8696044 =$) 444,132 inches is the convex ſurface.

Therefore the expence will be 1 £. 17 *s.*

SECTION V.

Of arched roofs.

When a building is covered with an arched roof, ſuch roofs are, either, Vaults, Domes, Salons, *or* Groins.

VAULTS, When the curved ſides of the roof ſpring from oppoſite or ſide walls, and meet in a right line over the middle of the building.

Such are the middle Iſles of moſt churches.

DOME, When the ſides of the arched roof ſpring from a circular or polygonal baſe, and meet in, or tend to, a point directly over the centre of that baſe.

SALON or SALOON, When a flat roof, or ceiling, is joined to the ſide walls by arcs of ſome one curve.

GROINS, When a vaulted roof is interſected by other vaults.

Of vaulted roofs.

They are generally of one of theſe three ſorts.

$$\left.\begin{array}{l}\text{1. }\textit{Circular},\\ \text{2. }\textit{Elliptic},\end{array}\right\}\text{ when the arch is ſome part}$$

$$\text{of the periphery of}\left\{\begin{array}{l}\text{a circle.}\\ \text{an ellipſe.}\end{array}\right.$$

3. *Gothic*, When the arch conſiſts of two circular arcs meeting in a point directly over the middle of the breadth, or ſpan, of the arch.

PROPOSITION XXXIII.

To find the ſolid content of circular, elliptic, or gothic, vaulted roofs.

RULE.

Multiply the area of one end by the length, and the product will be the ſolid content.

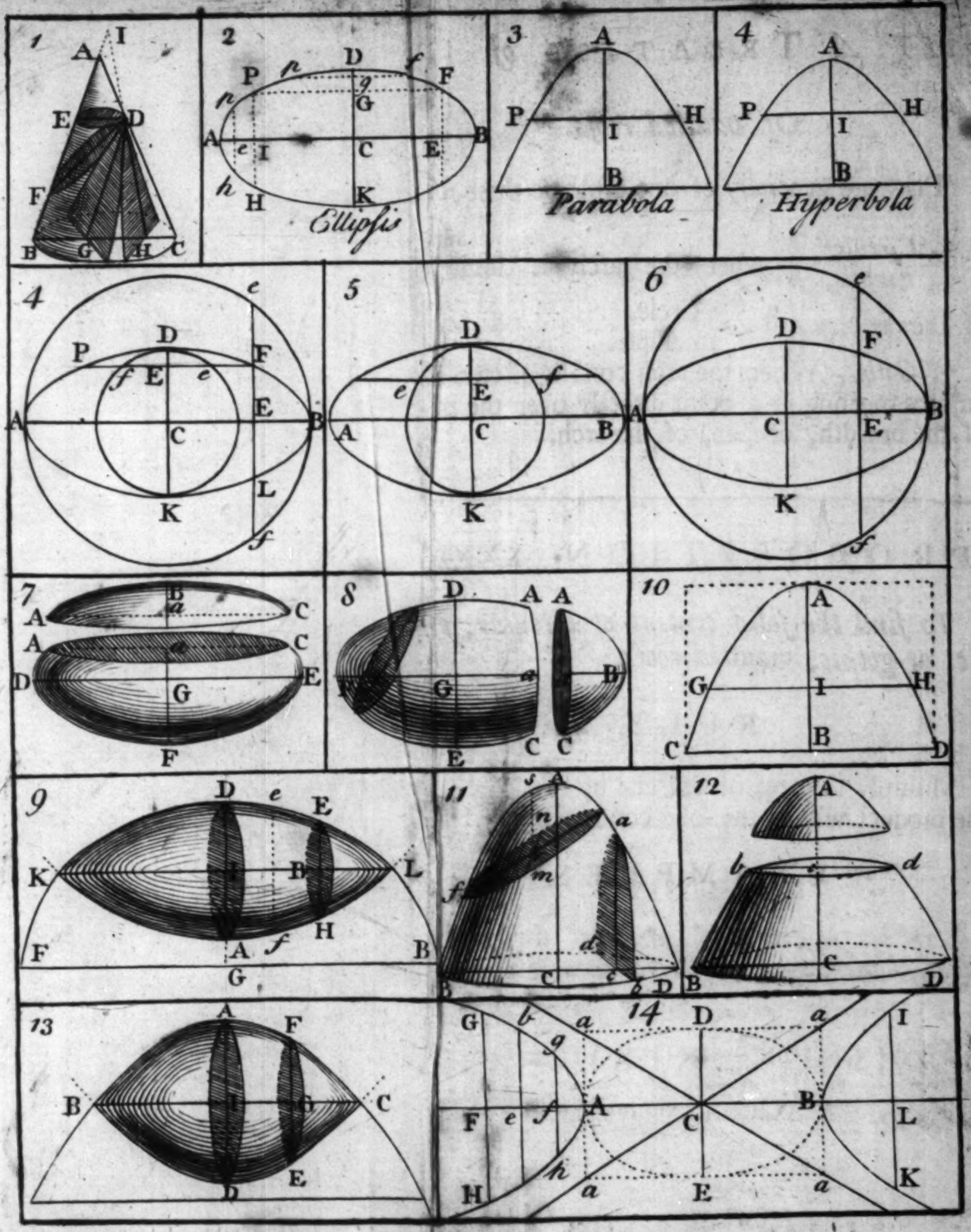
EXAMPLES.

I. *What is the ſolid content of a ſemicircular vault, whoſe ſpan is* 40 *feet, and length* 120 *feet?*

Then $\left(3{,}1416 \times \left.\overline{\frac{40}{2}}\right|^{2} \times \frac{1}{2} \times 120 = \right)$ 75398,4 ſolid feet, is the content required.

II.

Pl III Pa 311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II. *In an elliptic vault whoſe ſpan is* 40 *feet, height* 12, *and length* 80 *feet: Required the ſolid content?*

Then $(40 \times 12 \times 0{,}7854 \times 80 =)$ 30159,36 ſolid feet is the content ſought.

III. *What is the ſolid content of a gothic vault, whoſe ſpan is* 48 *feet, the chord of its arch,* 48 *feet, the diſtance of the arch from the middle of the chord is* 18 *feet, and the length of the vault is* 18 *feet?*

Now 1272,7874 is the area of the two circular ſegments, (ſee p. 163.)

And $\sqrt{\overline{48}^2 - \overline{\frac{48}{2}}^2} = 41{,}57$ is the height of the arch.

Then $\left(\overline{1272{,}7874 + 41{,}57 \times \frac{48}{2}} \times 60 =\right)$ 136228,044 is the ſolidity required.

If the ſolidity of the materials in either of theſe arches was required?

RULE.

From the ſolid content including the arch, take the ſolid content of the void, and the remainder will be the ſolidity of the arch.

PROPOSITION XXXIV.

To find the concave, or convex ſurface of circular, elliptic, or gothic vaulted roofs.

RULE.

Multiply the length of the arch by the length of the vault, and the product will be the ſuperficies required.

NOTE.

If the convex ſurface of the vault is required; it will be moſt ready and accurate, to ſtretch a ſtring over the convexity of the vault, and this ſtring meaſured gives the length of the curve.

But for the concave ſurface, this method is not ſo applicable, and the length of the arch muſt be found, from proper dimenſions, as ſhewn in Prop. XX. p. 158. or in Prop. II. p. 258.

Of domes.

There may be a great variety of domes, ariſing from the figure of their baſe, their height, and the nature of their curved ſides: But as the moſt common in uſe, are ſuch whoſe baſe is either a circle or a regular polygon, and whoſe curved ſides are circular, or elliptic quadrantal arcs, therefore theſe only, will here be treated of.

PROPOSITION XXXV.

To find the ſolid content of a dome, whoſe height and the dimenſions of its baſe are known.

RULE.

Multiply the area of the baſe by two thirds of of the height, and the product will be the ſolid content.

Ex. I. *What is the ſolid content of a ſpherical dome, the diameter of whoſe circular baſe is* 60 *feet?*

Now $\overline{60}^2 \times 0{,}7854 = 2767{,}44$ is the area of the baſe.

And 30 is the height.

Then $\left(2767{,}44 \times \frac{2 \times 30}{3} =\right)$ 55348,8 cubic feet is the ſolid content.

Ex. II. *In a hexagonal ſpherical dome, one ſide of the baſe is 20 feet: Required the ſolid content?*

Now (by tab. I. p. 144. $0,8660254 \times 20 =$) 17,320508 is the radius of the inſcribed circle, or height of the dome.

Alſo (by tab. I. p. 144. $2,5980762 \times \overline{20}\vert^2 =$) 1039,23048 is the area of the baſe.

Then ($1039,23048 \times 17,320508 \times \frac{2}{3} =$) 12000 ſolid ſeet is the content.

Ex. III. *A maſon has built an octagonal elliptic dome, whoſe inſide height is* 60 *feet; the diameter of its greateſt inſcrib'd circle is* 40 *feet; the thickneſs of the ſtone work at the bottom is* 8 *feet, and at the top is* 4 *feet: What will be the expence of this dome at* 12*s. a foot ſolid?*

Now $\left(\text{by tab. III. p. 145. } 3,3137084 \times \overline{\frac{40}{2}}\Big\vert^2 =\right)$ 1325,48336 is the area of the inner baſe.

And ($1325,48336 \times 60 \times \frac{2}{3} =$) 53019,3344 is the ſolid content of the void.

Alſo (by tab. III. p. 145. $3,3137084 \times \overline{28}^2 =$) 2597,9473856 is the area of the outward polygon.

And ($2597,9473856 \times 64 \times \frac{2}{3} =$) 110847,08845 is the ſolid content of the dome.

Then ($110847,08845 - 53019,3344 =$) 57827,75405 is the cubic feet of ſtone work, which will amount to 34696£. 13*s.* 0½*d.*

PROPOSITION XXXVI.

To find the ſuperficial contents of a ſpherical dome.

RULE.

Twice the area of the baſe is the ſuperficial contents required.

EXAMPLE

What will the painting of a hexagonal ſpherical dome come to at 1s. a yard; each ſide of the baſe being 20 feet?

Now (by tab. I. p. 144. $2,5980762 \times \overline{20}|^{2} =$) 1039,23048 is the area of the baſe.

Then 2078,46096 is the ſuperficial content.

And the expence will be about 103£. 18*s.* 6*d.*

In elliptic domes, it will be near enough for practice, to work by the following,

RULE.

To half the diameter at the baſe add the height, the ſum multiplied by 1,5708 will give the ſuperficial content nearly.

Examples to this, are eaſily ſupplied.

Of Salons.

This ſort of roofing or ceiling is generally uſed to cover ſuch buildings or rooms, whoſe plan is either rectangular, circular, or a regular polygon: And the curved parts are circular or elliptic quadrantal arcs: Alſo, the ſides of the flat part of the ceiling, are each alike equidiſtant from the walls of the room. In what follows, by flat ceiling, underſtand the middle or flat part of the Salon.

PROPOSITIN XXXVII.

To find the ſolid content of a Salon, the figure and ſides of the flat ceiling; the ſides of the room, the height of the arch, and its projection from the wall being reſpectively known.

RULE.

Multiply the height of the arch, its projection, one fourth of the perimeter of the ceiling, and 3,1416 continually; call the product A.

From $\left\{\begin{matrix}\text{a ſide}\\ \text{diam.}\end{matrix}\right\}$ of the room, take a $\left\{\begin{matrix}\text{like ſide}\\ \text{diamet.}\end{matrix}\right\}$ of the ceiling; the ſquare of the remainder multiplied by the proper factor; (page 144. 143.) the product multiplied by two thirds of the height of the arch, call B.

Multiply the area of the flat ceiling by the height of the arch, the product added to the ſum of A and B, will give the ſolid content required.

Ex. I. *What is the ſolid content of a Salon with a circular quadrantal arch of 2 feet radius, ſpringing over a rectangular room of 20 feet long and 16 feet wide.*

Here the flat part of the ceiling is 16 by 12 ſeet.

Then $\left(2 \times 2 \times \frac{16 \times 2 + 12 \times 2}{4} \times 3{,}1416 =\right)$ $175{,}9296 = A.$

And $(\overline{20 - 16}^{2} \times 1{,}0000000 \times 2 \times \frac{2}{3} =)\ 21{,}3$ $= B.$

Therefore $16 \times 12 \times 2 + 175{,}9296 + 21{,}3$ $= 581{,}2629$ ſolid ſeet, is the content ſought.

Ex. II. *A circular building of 40 feet diameter, and 25 feet high to the ceiling, is covered with a Salon whoſe circular arch is 5 feet radius: Required the capacity of that room in cubic feet?*

Now $(40 - 5 \times 2 =)\ 30$ is the diameter of the ceiling.

And $(30 \times 3{,}1416 =)\ 94{,}248$ is the circumference thereof.

Then $\left(5 \times 5 \times \frac{94{,}248}{4} \times 3{,}1416 =\right)$ 1850,-
$55948 = A.$

And $(\overline{40 - 30}^{2} \times 0{,}7854 \times 5 \times \frac{2}{3} =)\ 261{,}8$ $= B.$

Alſo $(\overline{30}^{2} \times 0{,}7854 \times 5 =)\ 3534{,}3$ for the cylindric part of the Salon.

Hence the ſolid. of the Salon $= 5646{,}65948.$

And $(\overline{40}^{2} \times 0{,}7854 \times 20 =)\ 25132{,}8$ is the ſolidity of the cylindric part of the room.

Then 30779,45948 is the ſolid content of that room.

Ex. III. *A gentleman who has in his garden a hexagonal ſummer-houſe each ſide within being 7 feet, and the walls 18 inches thick; orders a maſon to cover this with a pyramidal ſtone roof, whoſe ſides ſhall riſe 6 inches within the outſide of the walls, and its height be equal to the radius of the circle inſcrib'd in the pyramids baſe; alſo the inſide to be wrought in an elliptic ſalon, fitted to the ſpan of the room, the height of the arch to be 2 feet, and its projection 3 feet: But inſtead of the flat part of the ceiling, he will have a ſpherical hexagonal dome to ſpring from the upper extremities of the other arch: What will this roof come to, at 12s. a foot ſolid?*

Now $(7 \times 0{,}8660254 =)$ 6,0621778 is the radius of the circle inſcrib'd in the room. (tab. I. p. 144.)

Then 7, 0621778 is the height of the pyramid; and alſo, is the radius of the circle inſcrib'd in its baſe.

Hen. $\left(\overline{7{,}0621778}|^{2} \times {}^{*}3{,}4641016 \times \frac{7{,}0621778}{3} =\right)$ 406,710411 is the ſolidity of the pyramid.

(* Tab. III p. 145.)

Again $(6{,}0621778 - 3 =)$ 3,0621778 is the radius of a circle inſcrib'd in the flat ceiling of the ſalon.

Then $(3{,}0621778 \times {}^{*}1{,}1547005 =)$ 3,5358981 is the ſide of that ceiling.

(* Tab. III. p. 145.)

And $\left(2 \times 3 \times \frac{3{,}5358981\ 6 \times 6}{4} \times 3{,}1416 =\right)$ $99{,}9753962 = A$.

Alſo $(\overline{7 - 3{,}5358981}^{|2} \times {}^{*}2{,}5980762 \times 2 \times \tfrac{2}{3} =)$ $41{,}569224 = B$.

* (Tab. I. p. 144.)

Now $(\overline{3{,}0621778}^{|2} \times {}^{*}3{,}4641016 =) 32{,}4826471$ is the area of the flat part of the ceiling.

(* Tab. III. 145.)

Then $(32{,}4826471 \times 2 + A + B =)$ 206,5098144 is the ſolid content of the ſalon.

But 3,0621778, the radius of the ceiling, is alſo the height of the dome.

And $(32{,}4826471 \times 3{,}0621778 \times \tfrac{2}{3} =)$ 66,3117605 is the ſolid content of the dome.

Therefore 272,821575 is the ſolid centent of the excavation or hollow of the roof.

And 133,888836 is the ſolidity of the ſtone work.

Hence the expence will be 80 £. 6 *s.* 8 *d.*

PROPOSITION XXXVIII.

To find the ſuperficial content of a ſalon; the figure and ſides of the flat ceiling, the ſides of the room, the height of the arch and its projection from the wall being reſpectively known.

RULE.

1ſt, Find the area of the flat part of the ceiling.

2d, Find the convex ſurface of a cylinder or cylindroid, whoſe length is equal to one fourth of the perimeter of the ceiling, and its diameters equal to twice the height and twice the projection of the arch.

3d, Find the ſuperficial content of a dome of the figure of the arch, and whoſe baſe is either a ſquare, or a figure ſimilar to that of the ceiling; the ſide being equal to the difference of a ſide of the room and a ſide of the ceiling.

The ſum of theſe three articles will give the ſuperficial content ſought.

Note, In a $\left\{\begin{array}{c}\text{rectangular}\\ \text{circular}\\ \text{regul. polygonal}\end{array}\right\}$ room, the baſe of the dome will be a $\left\{\begin{array}{l}\text{ſquare}\\ \text{circle}\\ \text{like polygon.}\end{array}\right.$

Examples to this prop. are eaſily ſupplied.

Of Groins.

Theſe arches or roofs may be conſidered, as ariſing from the interſections of ſegments of circular cylinders, or elliptical cylindroids, cut off by planes parallel to their axes.

There may be a great variety of Groins produced, but in this place, no other will be conſidered, but thoſe whoſe interſections are at right angles; and of theſe, only ſuch as moſt commonly occur, and are formed.

I. By two circular equal ſemicylinders; and called circular groins.

II. By two elliptical equal ſemicylindroids; either on the tranſverſe or conjugate axes; and called elliptical groins.

In either caſe, the groin arches ſpring over a ſquare baſe.

PROPOSITION XXXIX.

To find the ſolid content of the vacuity form'd by a groin arch; either circular or elliptical; the ſide of the ſquare baſe, and the height of the groin being known.

RULE.

Multiply the area of the baſe by the height, the product multiplied by 0,9041295 will give the ſolid content required.

Ex. *What is the ſolid content of the vacuity form'd by a circular groin, one ſide of its ſquare baſe being* 12 *feet?*

Now (12 × 12 =) 144 is the area of the baſe.

And (144 × $\frac{12}{2}$ =) 864 is the product of the area of the baſe by the height.

Then (864 × 0,9041295 =) 781,167888 is the ſolid content required.

Ex. II. *What is the ſolid content of the vacuity form'd by an elliptical groin; one ſide of its ſquare baſe being* 20 *feet, and the height* 6 *feet?*

Then (20 × 20 × 6 × 0,9041295 =) 2169,9108 is the ſolid content required.

PRO-

PROPOSITION XL.

To find the concave ſuperficies of a circular groin arch; the ſide of the ſquare baſe being known.

Rule. Multiply the area of the baſe by 1,1415923, and the product will give the ſuperficies required.

This rule may be uſed for elliptical groins, the error being too ſmall to be regarded in practice.

Ex. *What is the curve ſuperficies of a circular groin arch; one ſide of its ſquare baſe being* 12 *feet?*

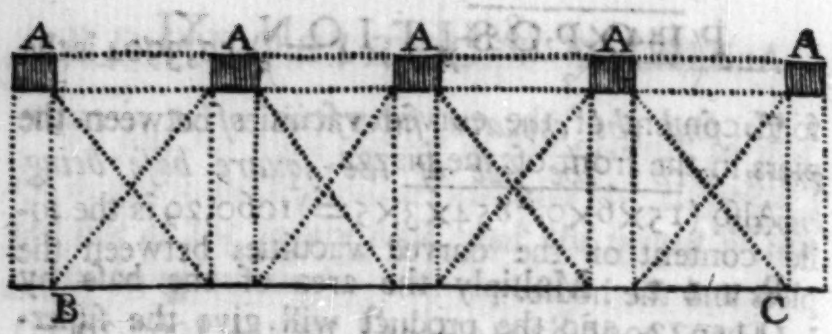
Then (12 × 12 × 1,1415923 =) 164,3892912 is the ſuperficies required.

Either of theſe rules, may be applied, in practice to groins contain'd under circular or elliptical ſegments, or on any rectangular baſe; the error, in ſome caſes, being leſs than thoſe, which frequently ariſe in taking the dimenſions of the ſame work by different perſons.

In meaſuring of work where there are many groins in a range, as a colonade, cloyſter, piazza, *&c.*; the cylindrical pieces between the groins, and on their ſides muſt be computed ſeparately: And to find the ſolidity of the brick or ſtone work which form the groin arches obſerve the following,

Rule. Multiply the area of the baſe, by the height, including the thickneſs of the work over the top of the groin, this product leſſened by the ſolid content, found by the former rule, will leave the ſolidity of the work.

EX.



Ex. *Let the figure above, repreſent part of a groin'd piazza, joining to the wall* BC *of a houſe; where the piers* A *next the ſtreet, are each* 3 *feet broad,* 2 *feet thick, and* 7 *feet high; the ſpan of the intermediate arches, are* 12 *feet each; the breadth of the piazza* 15 *feet in the clear; the height of the groins, above the piers,* 6 *feet: Now ſuppoſe the crown of the arch is* 2 *feet thick, and the upper ſurface made level, by working the ſpandrils up ſolid; what will the bill come to, at* 1 *s. a foot ſolid for the brick-work, and* 1 *s. a yard ſuperficies for the ſtucco plaiſtering?*

Now $(12\times4+5\times3=)$ 63 feet is the length of the piazza.

$(15+2=)$ 17 is the breadth.

$(6+2=)$ is the height of the groin-work, including the thickneſs thereof over the crown.

And $(63\times17\times8=)$ 8568 is the ſolidity of the roof, including the ſolid content of all the vaults.

Alſo $(3\times2\times7\times5=)$ 210 is the ſolidity of the piers.

Then $(8568+210=)$ 8778 is the ſolidity of the work, including the deductions.

Again $(15\times12\times6\times0,9041295\times4=)$ 3905,83944 is the ſolid content of the vacuities form'd by the 4 groins.

And $\left(\frac{\overline{144\times0{,}7854}}{2}\times2\times4=\right)452{,}3904$ is the ſolid content of the curved vacuities between the piers in the front of the piazza.

Alſo $(\overline{15\times6\times0{,}7854}\times3\times5=)1060{,}29$ is the ſolid content of the curved vacuities between the piers and the houſe.

Then $(3905{,}83944+452{,}3904+1060{,}29=)$ $5418{,}51984$ is the ſolid content of all the deductions.

Therefore $(8778-5418{,}51984=)3359{,}48016$ is the ſolid feet in all the work; which amounts to 167,974 £.

Again $(\overline{3+3+2+2}\times7\times5=)350$ is the ſuperficies of the 5 piers.

And $(\overline{63\times8}-\overline{0{,}7854\times12\times6}\times4=)277{,}8048$ is the ſurface the front, above the piers.

Alſo $(\overline{15\times12\times1{,}1415923}\times4=)821{,}946456$ is the concave ſurface of the 4 groins.

And $(\overline{3{,}1416\times6\times2}\times4=)150{,}7968$ is the curve ſurface of the 4 arches between the piers.

Alſo $\left(\overline{\frac{\overline{15\div2}+6}{2}\times3{,}1416\times3}\times5=\right)318{,}087$ is the curve ſurface of the 5 arches between the piers and the houſe.

The ſum of all theſe ſuperficies is 1918,635, &c. feet.

Or 213,181 yards, amounting to 10,659 £.

Therefore the whole expence will be 178 £. 12 *s.* 8 *d.*

SEC-

SECTION VII.

Of regular ſolids.

A Sphere is ſaid to circumſcribe a ſolid, when the angular points touch the concave ſurface of that ſphere.

A Sphere is ſaid to be inſcrib'd in a ſolid, when the plane ſides touch the convex ſurface of that ſphere.

A regular ſolid, is a body contain'd under equal, regular and like planes; alike poſited, and equally diſtant from the centre.

There are only five regular ſolids.

I. The TETRAEDON, contain'd under four equal and equilateral plane triangles; forming four ſolid angles, each of three triangles; and having ſix linear edges, and twelve plane angles.

II. The HEXAEDRON, contain'd under ſix equal ſquares; forming eight ſolid angles, each of three ſquares; and having twelve linear edges, and twenty-four plane angles.

III. The **Octaedron**, contain'd under eight equal and equilateral plane triangles; forming ſix ſolid angles, each of four triangles; having twelve linear edges, and twenty-four plane angles.

IV. The **Dodecaedron**, contain'd under twelve equal, equilateral, and equiangular pentagons; forming twenty ſolid angles, each of three pentagons, and having thirty linear edges and ſixty plane angles.

V. The **Icosaedron**, contain'd under twenty equal and equilateral triangles; forming twelve ſolid angles, each of five triangles; and having thirty linear edges, and ſixty plane angles.

By the following table, the ſuperficies, ſolidities, radii of the circumſcribed and inſcrib'd ſpheres, and the linear ſides or edges, of any of the regular ſolids, may be readily computed.

Let

$$\text{Let}\left\{\begin{array}{l}S\\Z\\X\end{array}\right\}\text{repreſent the}\left\{\begin{array}{l}\text{linear ſide}\\\text{ſuperficies}\\\text{ſolidity}\end{array}\right\}\begin{array}{l}\text{of a}\\\text{regular}\\\text{ſolid.}\end{array}$$

When	Then	Tetraedron.	Hexaedron.
S=1	R=	0,6123724	0,8660254
	r=	0,2041241	0,5000000
	Z=	1,7320508	6,0000000
	X=	0,1178511	1,0000000
R=1	S=	1,6329932	1,1547006
	r=	0,3.......	0,5773503
	Z=	4,6188023	8,.......
	X=	0,5132002	1,5396006
r=1	S=	4,8989795	2,.......
	R=	3,.......	1,7320508
	Z=	41,5692192	24,.......
	X=	13,8564064	8,.......
Z=1	S=	0,7598357	0,4082483
	R=	0,4653025	0,3535534
	r=	0,1551008	0,2041241
	X=	0,0517003	0,0680413
X=1	S=	2,0395489	1,.......
	R=	1,1547006	0,8660254
	r=	0,4163417	0,5......
	Z=	7,2056240	6,.......

And $\left\{\begin{matrix}R\\ r\end{matrix}\right\}$ the radius of the $\left\{\begin{matrix}\text{circumſc.}\\ \text{inſcrib'd}\end{matrix}\right\}$ ſphere.

Octaedron.	Dodecaedron	Iſocaedron.
0,7071068	1,4012585	0,9510565
0,4082483	1,1135164	0,7557613
3,4641016	20,6457288	8,6602540
0,4714045	7,6631188	2,1816951
1,4142136	0,7136442	1,0514622
0,5773503	0,7946545	0,7946545
6,9282032	10,5146223	9,5745413
1,3	2,7851639	2,5361507
2,4494897	0,8980560	1,3231691
1,7320508	1,2584086	1,2584086
20,7846096	16,6508731	15,1621684
6,9282032	5,5502910	5,0540561
0,5372850	0,2200822	0,3398088
0,3799178	0,3083920	0,3231774
0,2193457	0,2450651	0,2568144
0,0731152	0,08168837	0,08560479
1,2848990	0,5072221	0,7710254
0,9080604	0,7107492	0,7332887
0,5245576	0,5648000	0,5827111
5,7191069	5,3116140	5,1483486

The uſe of the foregoing table will be ſufficiently illuſtrated by the ſolution of the following Caſes; wherein, to avoid repetitions: By a ſolidity, a ſuperficies, or a ſide, is meant the ſolidity, the ſuperficies, or, the linear ſide or edge of a regular ſolid: And, by a radius; is meant, the radius of the ſphere that can either circumſcribe, or be juſt contain'd in, that regular ſolid.

Let N repreſent the tabular number; correſponding to either ſolid, and, its ſide; its radius; its ſuperficies; its ſolidity; where S, R, *r*, Z or X, = 1.

CASE I.

A $\left\{\begin{array}{l}\textit{ſide}\\ \textit{radius}\end{array}\right\}$ *being given; to find a* $\left\{\begin{array}{l}\textit{radius}\\ \textit{ſide.}\end{array}\right.$

RULE.

Multiply the given $\left\{\begin{array}{c}\text{ſide}\\ \text{radius}\end{array}\right\}$ by N $\left\{\begin{array}{l}\text{radius, where S} = 1;\\ \text{ſide, where R, or, r} = 1\end{array}\right\}$ and the product will be the $\left\{\begin{array}{c}\text{radius}\\ \text{ſide}\end{array}\right\}$ required.

EX-

EXAMPLE I.

If the ſide of a dodecaedron is 2; *required the radii of the circumſcrib'd and inſcrib'd ſpheres?*

Under the name dodecaedron, and againſt R and r, where S = 1; are the numbers 1,4012585 and 1,1135164.

$$\text{Then}\left.\begin{matrix}(1{,}4012585\times2=)2{,}8025170=R\\(1{,}1135164\times2=)2{,}2270328=r\end{matrix}\right\}\text{required.}$$

EXAMPLE II.

If the radius of a ſphere, circumſcribing a dodecaedron, be 2,802517; *required the ſide of that dodecaedron; and the radius of its inſcrib'd ſphere?*

Againſt $\left\{\begin{matrix}S\\r\end{matrix}\right\}$ where R=1; and under dodecaedron, is $\left\{\begin{matrix}0{,}7136442\\0{,}7946545\end{matrix}\right\}$ which multiplied by 2,802517; gives $\left\{\begin{matrix}2\text{ - - - - - }=S\\2{,}2270326=r\end{matrix}\right\}$ required.

EXAMPLE III.

If the radius of a ſphere, inſcrib'd in a dodecaedron, is 2,2270328: Required the ſide of that dodecaedron, and the radius of the circumſcrib'd ſphere?

Againſt $\left\{\begin{matrix}S\\R\end{matrix}\right\}$ where r = 1; and under dodecadron, is $\left\{\begin{matrix}0,8980560\\1,2584086\end{matrix}\right\}$ which multiplied by 2,2270328; gives $\left\{\begin{matrix}2 - - - - - = r\\2,8025171 = R\end{matrix}\right\}$ required.

CASE II.

A $\left\{\begin{matrix}\textit{ſide}\\\textit{radius}\end{matrix}\right\}$ *being given; to find a ſuperficies.*

RULE.

Multiply the ſquare of the given $\left\{\begin{matrix}\text{ſide}\\\text{radius}\end{matrix}\right\}$ by N ſuperficies $\left\{\begin{matrix}\text{where S - - - -} = 1.\\\text{where R, or, r} = 1.\end{matrix}\right\}$ And the product will give the ſuperficies required.

EXAMPLE I.

What ic the ſuperficies of a dodecaedron whoſe ſide is 2?

Then $(20,6457288\times\overline{2}|^{2} =)$ 82,5829156 = Z required.

EXAMPLE II.

What is the ſuperficies of a dodecaedron inſcrib'd a ſphere whoſe radius is 2,802517 ?

Then $(10,5146223\times\overline{2,802517}|^{2}=)82,5829159$ $=$ Z required.

EXAMPLE III.

What is the ſuperficies of a dodecaedron, circumſcribing a ſphere, whoſe radius is 2,2270328 ?

Then $(16,6508731\times\overline{2,2270328}|^{2}=)82,5829156$ $=$ Z required.

CASE III.

A $\left\{\begin{matrix}\textit{ſide}\\ \textit{radius}\end{matrix}\right\}$ *being given; to find a ſolidity?*

RULE

Multiply the cube of the given $\left\{\begin{matrix}\text{ſide}\\ \text{radius}\end{matrix}\right\}$ by N ſolidity $\left\{\begin{matrix}\text{where } S=1\\ \text{where } R,\text{ or, } r=1\end{matrix}\right\}$ and the product will give the ſolidity required.

EXAMBLE I.

What is the ſolidity of a dodecaedron whoſe ſide is 2?

Then $(7,6631188\times\overline{2}|^{3}=)$ $61,3049504=$ X required.

EXAMPLE II.

What is the ſolidity of a dodecaedron inſcrib'd in a ſphere whoſe radius is 2,80257 ?

Then $(2,7851639\times\overline{2,80257}|^{3}=)61,3049499=$ X required.

EXAMPLE III.

What is the ſolidity of a dodecaedron, circumſcribing a ſphere whoſe radius is 2,2270328?

Then $(5,5502910\times\overline{2,2270328}|^{3}=)61,3049509$ $=$ X required.

CASE IV.

A ſuperficies being given; to find a $\left\{\begin{matrix}\textit{ſide}\\ \textit{radius.}\end{matrix}\right.$

RULE.

Multiply the ſquare root of the given ſuperficies, by N $\left\{\begin{matrix}\text{ſide}\\ \text{radius}\end{matrix}\right\}$ where $Z=1$; and the product will be the $\left\{\begin{matrix}\text{ſide}\\ \text{radius}\end{matrix}\right\}$ required.

EXAMPLE.

If the ſuperficies of a dodecaedron is 82, 5829152; required S, R, r?

I. $(0,2200822\times\sqrt{82,5829152}=)2,$ - - - - - $=S.$
II. $(0,3085920\times\sqrt{82,5829152}=)2,8025166=R$
III. $(0,2450651\times\sqrt{82,5829152}=)2,2270324=r.$

CASE V.

A ſolidity being given; to find a $\left\{\begin{matrix}\textit{ſide}\\ \textit{radius.}\end{matrix}\right.$

RULE.

Multiply the cube root of the given ſolidity, by the N $\left\{\begin{matrix}\text{ſide}\\ \text{radius}\end{matrix}\right\}$ where $X=1$; and the product will be the $\left\{\begin{matrix}\text{ſide}\\ \text{radius}\end{matrix}\right\}$ required.

EXAMPLE.

If the ſolidity of a dodecaedron is 61,3049504; *required S, R, r?*

I. $(0{,}5072221\times\sqrt[3]{61{,}3049504}=)2,\qquad=S$

II. $(0{,}7107492\times\sqrt[3]{61{,}3049504}=)2{,}8025168=R$

III. $(0{,}5648000\times\sqrt[3]{61{,}3049504}=)2{,}2270323=r.$

CASE VI.

A ſuperficies being given; to find a ſolidity.

RULE.

Multiply the given ſuperficies by its ſquare root; the product multiplied by the N^s ſolidity, where $Z=1$, will give the ſolidity required.

EXAMPLE.

What is the ſolidity of a dodecaedron, whoſe ſuperficies is 82,5829152?

Now $\sqrt{82{,}5829152}=9{,}0875142$.

And $82{,}5829152\times9{,}0875142=750{,}4734144$.

Then $750{,}4734144\times0{,}08168837=61{,}3049499=X$.

CASE VII.

A ſolidity being given; to find a ſuperficies.

RULE.

Divide the given ſolidity by its cube root; the quotient multiplied by N ſuperficies, where $X=1$, will give the ſuperficies required.

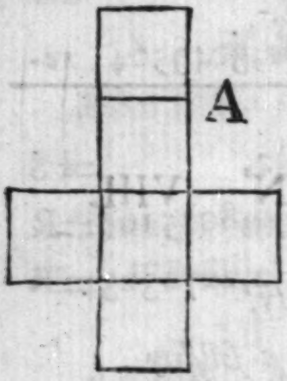
EXAMPLE.

What is the ſuperficies of a dodecaedron whoſe ſolidity is 61,3049504?

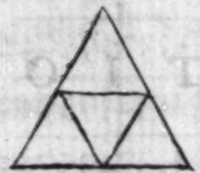
Now $\sqrt[3]{61{,}3049504}=3{,}943046$.

And $\frac{61{,}3049504}{3{,}943046}=15{,}5476122$.

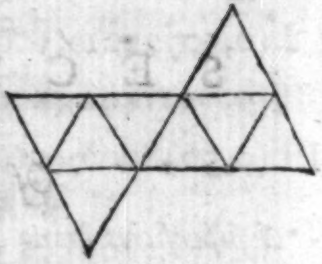
Then $15{,}5476122\times5{,}3116140=)\ 82{,}5829146=Z$.



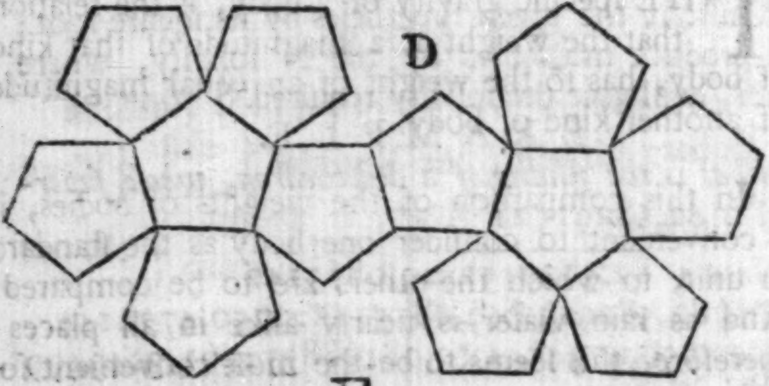
A



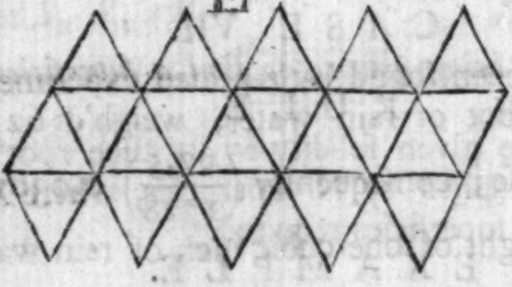
B



C



D



E

In five ſuch figures as theſe, made of paſt-board, or any other pliable ſubſtance; if the lines be cut half through, and the parts turned up and glued together, the figures will repreſent the five regular ſolids: *viz.* A, the hexaëdron; B, the tretraëdron; C, the octaëdron; D, the dodecaëdron; and E, the Icoſaëdron.

SEC-

SECTION VIII.

Of ſpecific gravity.

THE ſpecific gravity of a body, is the relation that the weight of a magnitude of that kind of body, has to the weight of an equal magnitude of another kind of body.

In this comparifon of the weights of bodies, it is convenient to conſider one body as the ſtandard or unit, to which the others are to be compared: And as rain water is nearly alike in all places; therefore, this ſeems to be the moſt convenient for a ſtandard.

It has been found by repeated experiments, that a cubic foot of rain water, weigh'd $62\frac{1}{2}$ pounds averdupoiſe; conſequently $\left(\frac{62,5}{1728}\right)$ 0,03616898 ℔ is the weight of one cubic inch of rain water.

The knowledge of the ſpecific gravities of bodies, is of great uſe in computing the weights of ſuch bodies as are too heavy or too unweildy to have their weight diſcover'd by other means.

A TABLE *ſhewing the ſpecific gravity, to*
And the weight of a cubic inch of each,

Bodies.	Sp. gravity.	Wt. ℔ averd.
Fine gold - - - -	19,640	0,7103587
Eng. gold coin -	19,520	0,7060185
Caſt gold - - - -	18,888	0,6828703
Quickſilver - - -	13,762	0,4976574
Lead - - - - - - -	11,313	0,4091696
Fine ſiver - - - -	11,091	0,4011501
Eng. ſilver coin -	10,629	0,3844400
Caſt ſilver - - - -	10,528	0,3807870
Copper - - - - -	8,769	0,3171658
Caſt braſs - - - -	8,104	0,2929832
Steel - - - - - - -	7,850	0,2839265
Bar iron - - - - -	7,764	0,2808159
Block tin - - - - -	7,238	0,2617901
Caſt iron - - - -	7,135	0,2580647
Loadſtone - - - -	5,106	0,1846788
Blue ſlate - - - -	3,500	0,1264914
Vein'd marble - -	2,702	0,0977286
Common glaſs -	2,600	0,0940393
Flintſtone - - - -	2,582	0,0933883
Portlandſtone - -	2,570	0,0929543
Freeſtone - - - -	2,352	0,0915788
Brick - - - - - - -	2,000	0,0723379
Ivory - Horn - } - - - -	1,832	0,0662606

Note, 7000 grains make 1 ℔ averdupoiſe.
And 5760 grains make 1 ℔ troy.
Therefore, the ℔ averd. : ℔ troy : : 700 : 576.

rain-water; of metals and other bodies:
in parts of a pound averdupoiſe.

Bodies.	Sp. gravity.	Wt. ℔ averd.
Brimſtone - - - -	1,800	0,0651042
Clay - - - - - - - -	1,712	0,0619213
Lignum Vitæ - -	1,327	0,0479862
Coal - - - - - - - -	1,255	0,0453921
Pitch - - - - - - -	1,150	0,0415943
Mahogany wood	1,063	0,0384475
Dry box wood -	1,030	0,0372530
Milk - - - - Sea water -	1,030	0,0372530
Rain water - - -	1,000	0,0361690
Bees wax - - - -	0,995	0,0359881
Dry oak - - - - -	0,915	0,0330946
Olive oil - - - -	0,913	0,0330222
Beech - - - - - - -	0,854	0,0308883
Dry elm - Dry aſh -	0,800	0,0289352
Dry wainſcot - -	0,747	0,0270182
Dry yellow fir -	0,657	0,0237630
Cedar - - - - - -	0,613	0,0221715
Dry white deal -	0,569	0,0205801
Cork - - - - - - - -	0,240	0,0186805
Air - - - - - - - - -	0,0012	0,0000434

Conſequently ℔ averd. mult. by $\frac{5,76}{7}$ gives ℔ troy.

And ℔ troy multi. by $\frac{700}{576}$ gives ℔ averd.

Note, $\frac{700}{576} = 1,215278$ nearly.

CASE I. *The linear dimenſions, or ſolidity, of a body being given; to find its weight.*

RULE. Multiply the cubic inches contain'd in that body, by the tabular weight correſponding to the name of the ſame kind; and the product will give the weight in pounds averdupoiſe.

EX. I. *What is the weight of a piece of oak, of a rectangular form, whoſe length is 56 inches; breadth 18, and depth 12 inches?*

Now ($56 \times 18 \times 12 =$) 12096 inches is the ſolidity.

Then ($12096 \times 0,0330946 =$) 400,3122816 ℔ is the weight required.

EX. II. *What is the weight of an iron ſhot, of 7 inches diameter.*

Now ($\overline{7}|^3 \times 0,5206 =$) 179,5948 inches is the ſolidity.

Then ($179,5948 \times 0,2580647 =$) 46,34706 ℔ is the weight required.

Ex. III. *What is the weight of an iron bomb-ſhell, of* 3 *inches thick; the greateſt diameter being* 16 *inches?*

Now $(16-\overline{3\times2}=)$ 10 is the diameter of the concavity.

And $(\overline{16}|^{3}\times0,5236=2144,6656$.

Alſo $(\overline{10}|^{3}\times0,5236=523,6$.

Then $(2144,6656-523,6=)$ 1621,0656 is the ſolidity of the ſhell.

Therefore $(1621,0656\times0,2580647=)$ 418, 3398 ℔ is the weight required.

Ex. IV. *Required the weight of one of the portland keyſtones, to the middle arch of* Weſtminſter-*bridge: The diameter of the arch being* 76 *feet; the height of the keyſtone* 5 *feet; the chord of its greateſt breadth, to the front of the arch,* 3 *feet* 4 *inches; and its depth, in the arch,* 4 *feet?*

Now $(76+5:3,\dot{3}::76:)$ 3,127572 is the chord of the leaſt breadth of the keyſtone.

Here the chords and their arcs may be ſuppoſed equal; the exceſs, in the greateſt arc, not being more than about $\frac{1\frac{1}{2}}{5000}$ part of an inch.

Now $(\overline{3,127572\times4+3,\dot{3}\times4}\times\frac{3}{2}\times\frac{5}{3}=)$ 64, 60905 feet is the ſolidity of the keyſtone.

Then $(64,60905\times1728\times0,0929543=)$ 10377, 83062 ℔.

Or 4 tons, 12 hund. 2 quart. 17,83 ℔ is the weight required.

CASE II. *The weight of a body being given; to find the ſolidity.*

RULE. Divide the given weight, in pounds averdupoiſe, by the tabular weight, correſponding to the name of the ſame kind; and the quotient will be the ſolidity in cubic inches.

EX. I. *What will a block of marble, weighing 8 tons, 14 C. wt. come to, at 6 s. a foot ſolid.*

Now 8 t. 14 c. = 19488 ℔.

Then $\left(\frac{19488}{0,0977286} \div 1728 =\right)$ 115,4 are the cubic feet.

And $(115,4 \times 0,3$ £. =) 34 £. 12 *s.* 5 *d.* is the coſt.

EX. II. *What is the diameter of an iron ſhot, weighing 42 pounds averd?*

Now $\left(\frac{42}{0,2580647} =\right)$ 162,7499 are the cubic inches.

Then $\left(\sqrt[3]{\frac{162,7499}{0,5236}} =\right)$ 6,7743 is the diameter required.

How many inches, will a cubic foot of dry elm, ſink in common water?

Answer. It will ſink, until a bulk of water equal to the part immerſed, be equal in weight to that of all the elm.

Now (1728 × 0,0289352 =) 50,0000256 ℔ is the weight of a foot of elm; and alſo, the weight of the water diſplaced.

And $\left(\frac{50{,}0000256}{0{,}036169}=\right)$ 1382,4 are the cubic inches immerſed.

Then $\left(\frac{1382{,}4}{144}=\right)$ 9,6 inches, is the depth to which a cubic foot of elm will ſink in common water.

How much weight is juſt neceſſary, to immerſe a cubic foot of yellow firr in ſea water?

Answer. So much weight, as is equal to the difference between the weights of a cubic foot of ſea water, and that of firr.

Now (0,037253 × 1728 =) 64,373184 ℔ is the weight of a cubic foot of ſea water.

And (0,023763 × 1728 =) 41,062464 ℔ is the weight of a cubic foot of dry firr.

Then (64,373184 − 41,062464 =) 23,31072 ℔ muſt be added to a cubic foot of firr, to immerſe it in ſea water.

Or. *The difference between the ſpecific weights, multiplied by the cubic inches in the body to be immerſed, will give the additional weight.*

Thus (0,037253 − 0,023763 =) 0,01349 × 1728 = 23,31072 ℔ as found before.

How many ſolid feet of yellow firr, laſh'd to a braſs cannon of 56 C. wt. will be ſufficient to keep it afloat at ſea.

Now 56 C. wt. = 6272 ℔ averdupoiſe.

And $\left(\frac{6272}{0,2929832}=\right)$ 21047,371 are the cubic inches of braſs.

Hence $(21047,371 \times 0,037253 =)$ 797,4887 ℔ is the weight of a bulk of ſea water, equal to that of the cannon.

Therefore $(6272 - 797,4887 =)$ 5474,5113 ℔ of the cannon is to be buoy'd up by the firr.

And the weight to be buoy'd up, divided by the difference of the ſpecific weights, of the body which is to buoy, and the fluid in which it is to float, will give the ſolidity of the buoying body.

That is, $\left(\frac{5474,5113}{0,037253-0,023763}=\right)$ 405819,936 cubic inches of firr will ſuſtain the cannon.

But 405819,936 cubic inches = 234,8495 feet ſolid.

Conſequently 12 pieces of firr timber, each of about a foot ſquare and 20 feet long, will ſuffice to keep ſuch a piece of cannon afloat in ſea water.

How thick muſt be the metal, of a concave copper ball, 6 inches in its outſide diameter, ſo as to ſink to its centre in common water?

Now ($\overline{6}|^3 \times 0,5236 =$) 113,697 cubic inches, is the ſolidity of that ſphere.

And $\left(\frac{113,0976}{2} =\right)$ 56,5488 cubic inches to be immerſed.

Or cubic inches of water to be remov'd.

Therefore ($56,5488 \times 0,036169 =$) 2,0453 ℔ is the weight of the water diſplaced, or the weight of the copper ball.

Conſequently $\left(\frac{2,0453}{0,3171658} =\right)$ 6,44867 are the cubic inches of copper in that ball.

But ($\overline{6}|^2 \times 0,7854 \times 4 =$) 28,2744 ſquare inches, is the ſuperficies of the ball.

Then $\left(\frac{6,44867}{28,2744} =\right)$ 0,2881 part of a linear inch, is the thickneſs (nearly) required.

SCHO-

SCHOLIUM.

The ſolidity, or weight, of any body, however irregular, may be very exactly determin'd, as follows.

Into an uniform veſſel, (whoſe horizontal ſections may be readily computed,) pour ſo much water, as may be judg'd neceſſary to cover the body (whoſe ſolidity is required) when immers'd therein; and note the height of the fluid in the veſſel: Immerſe the body, and note how high the fluid has ris'n: Then the ſolid content of the additional ſpace occupied by the fluid, on account of the immerſed body, will be equal to the ſolidity of that body: And conſequently, its weight is readily known.

And thus, may the ſolidity of ſtatues, *&c.* be very exactly computed.

The following remarks are here added, becauſe they may be uſeful to ſome perſons.

I. Of *Newcaſtle* coal, 60 ſolid feet are equal to 1 chaldron.

Therefore; divide the ſolid content in feet, of a vault, cellar, or other place by 60, and the quotient will ſhew how many chaldrons of coals that place will hold.

II. The contents of a *Wincheſter* buſhel when heaped, is in proportion to the contents of the ſame buſhel when ſtruck, as 4 to 3.

Therefore the conical heap, is one third of the cylindrical contents.

III. Straw is generally ſold in truſſes, each of 36 ℔ averdupoize, and 36 truſſes make a load.

IV. Hay is generally ſold by the load, containing 36 truſſes, each of 56 ℔, or half a hundred weight; ſo that a load of hay weighs 18 hundred weight.

Truſſes of hay cut out of a rick, are of various dimenſions, according to the goodneſs of the hay, and the care taken in laying it up.

The largeſt truſſes will ſeldom exceed 14 ſolid feet; and the ſmalleſt will rarely give leſs than 7 feet ſolid, and yet each of them weigh half a hundred weight.

But upon a mean, a truſs may be reckoned at 10 or 11 feet ſolid; and a load will contain about 360 or 400 feet.

In ricks of good hay, the farmers commonly cut their truſſes of five ſpans in length, three and a half ſpans in breadth, and one and a half, or two ſpans in thickneſs; or try a thickneſs that will, with the other dimenſions, make half a hundred weight.

Hay-ſtacks ſtand either on a rectangular, or on a circular baſe; and are compoſed of two tapering ſolids; the lower one, with its leaſt end downwards; and the higher one, with its leaſt end uppermoſt; ſo that the thickeſt part of the rick is the end common to both ſolids: The height of the lower ſolid is commonly from about 6 to 10 feet; and the height of the upper one, from about 24 to 18 feet.

To find the number of loads contain'd in a circular hay-rick.

RULE.

1 Multiply twelve times the height of the lower ſolid, by the ſquare of half the ſum of its greateſt and leaſt diameters.

2. Multiply five times the height of the upper ſolid, by the ſquare of its greateſt diameter.

3. The ſum of theſe two products, multiplied by 654; cut of the four right-hand figures, and thoſe to the left-hand will be ſolid feet.

4. The ſolid feet divided by 400, or by 360, will give the loads.

EXAMPLE.

How many loads are contained in a round hay-rick, whoſe lower ſolid is 8 feet high, 24 feet in the leſſer diameter, and 30 feet in the greater diameter; and the upper ſolid is 20 feet high?

Add $\begin{cases} 30 \\ 24 \end{cases}$

2)54

Mul. $\begin{cases} 27 \\ 27 \end{cases}$

189

54

729

96

4374

6561

69984

Mul. $\begin{cases} 30 \\ 30 \end{cases}$

Mul. $\begin{cases} 900 \\ 100 \end{cases}$

Add $\begin{cases} 90000 \\ 69984 \end{cases}$

159984

Mul. $\begin{cases} 159984 \\ 654 \end{cases}$

639936

799920

959904

4,00)10462,9536 ſo. fe.

26,$\frac{1}{6}$ loads nearly.

To find the loads contain'd in a rectangular hay-rick.

RULE.

1. In the lower ſolid; multiply the leſſer length by the leſſer breadth; and the greater length by the greater breadth; add the products together, multiply the ſum by half the height, reſerve the product.

2. In the upper ſolid; multiply the length of the greater end by its breadth; divide the product by twelve; multiply the quotient by ſeven times the height; reſerve the product.

3. The ſum of theſe two reſerved products will give the number of ſolid feet; which divided by 360 or 400, will give the loads.

EXAMPLE.

In a rectangular rick, the lower ſolid is 10 *feet high,* 36 *feet long and* 12 *feet wide at the leſſer end,* 40 *feet long and* 18 *feet wide at the greater end; and the upper ſolid* 24 *feet high: What is the rick worth at* 30 *ſhillings a load?*

Mul. $\left\{\begin{array}{r}36\\12\end{array}\right.$

Add $\left\{\begin{array}{r}432\\720\end{array}\right.$

1152

5

Add $\left\{\begin{array}{r}5750\\10080\end{array}\right.$ 1ſt prod.

4,00(15830

39 ½ loads.

Mul. $\left\{\begin{array}{r}18\\40\end{array}\right.$

720

Mul. $\left\{\begin{array}{r}18\\40\end{array}\right.$

12)720

Mul. $\left\{\begin{array}{r}60\\7\end{array}\right.$

Mul. $\left\{\begin{array}{r}420\\24\end{array}\right.$

1680

840

10080 2d produ.

Therefore the rick is worth 59 £. 5 *s.*

The reader muſt not expect, by theſe rules to compute the contents of hay-ſtacks, as accurately as the other ſolids in this work are computed by their reſpective rules; but he will by theſe means be able to value the ſtock of hay in ricks as nearly, perhaps, as by any other method; if regular methods for theſe things be already extant.

FINIS.

E R R A T A.

Page 33. line 7. for 9ʒ, read 0,ʒ; p. 78. l. 23. f. by the root 452. r. by thrice the root 452; p. 96. l. 9. f. of an arc. r. of that arc; p. 102. l. 6. f. 6. r. 16; p. 119. l. 9. of the work, f. 255. r. 2550; p. 121. l. 11. f. 172. r. 1720; p. 163. l. 17. f. = A, r. =Area; p. 165. l. 12. f. add its, r. add the ſquare of its; l. 12. f. ſquare of, r. ſquare root of; p. 193. in the latter part of the operation, f. 7. 2. 3. r. 8. 0. 7. for 7. 3. 9. 1, r. 7. 4. 6. 5; f. 153. 7. 11, r. 153. 7. 2; p. 211. l. 6. f. greater of the hoof, r. greater hoof; p. 225. l. 14. f. 242,ʒ, r. 249,ʒ; p. 245 l. 6. f. ,5792, r. 3792; p. 246. l. 5. f. 3361, r. 33651; p. 249. l. 11, f. 5,7, r. 3,7; p. 262. l. 20. f. 44. r. 24; p. p. 263. l. 3. f. PE. r. PF; p. 281. l. laſt but two, f. 734968, r. 7349968; p. 290. l. 7. f. $+\frac{2}{3}d$, r. $\times\frac{2}{3}d$; p. 294. l. 7, add Pl. III. Fig. 12; p. 305. l. 11. f. $\times\overline{\frac{72}{2}}\Big|^{2}$ r. $+\overline{\frac{72}{2}}\Big|$; p. 310. l. 8. f. ſolidity, r. ſuperficies; p. 326. l. 16. f. =r, r. =S.

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